

DSP Notes: Aliased Signals
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Problem:

Given a signal at f_0 (in Hz) that will suffer aliasing. What is the aliased frequency g_0 in Hz? Assume the signal is sampled and then reconstructed with an ideal A/D (with no anti-aliasing filter) and D/A. Use S for the sample rate. *Recall from the model of sampling that only signals within the principle range are output from the D/A.*

Procedure:

- 1) Check if aliasing will occur: **If $S < 2 f_0$ Then Aliasing Occurs**
- 2) Define $g_0 = f_0 - m S$, where m is an integer
- 3) Select m such that: $-S/2 \leq g_0 \leq S/2$

Variations on Procedure:

When working with digital frequencies, $G_0 = F_0 - m$, m is an integer.
Or, adjust the (radian) digital frequency Ω_0 by $2 m \pi$.

Justification:

- Given $x(n) = \cos(2 \pi f_0 n / S)$
- $= \cos(2 \pi f_0 n / S + 2 \pi m n)$, cosine periodic and m, n integers
- $= \cos(2 \pi f_0 n / S + 2 \pi m n S / S)$
- $= \cos\{2 \pi (f_0 + m S) n / S\}$, factoring

- So, the sequence $x(n)$ was unchanged by shifting the original analog frequency by some integer multiple of S .

- From the sampling model, we know about spectral replication that occurs during sampling. We also know that the output analog frequency is constrained to the principle range, by the ideal D/A.

- Hence the aliased output frequency is given by the above procedure, which selects an analog frequency in the principle range.