Filter Design by Pole/Zero Placement
For Band Pass and Notch Filters

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This type of filter is appropriate for high-Q band pass or notch filters. In this method, we will 'throw darts at the Z plane' to locate our poles and zeros. Filter parameters such as the amounts of attenuation or ripple in the stop band are not formal inputs to the design procedure. Rather these characteristics may be assessed after an initial pass through the design, and then adjusted (indirectly) via an iterative process.

Design Procedure:

1. (*Filter Specification*) Determine the center frequency, f_0 (Hz) and the sample rate, S. If a desired value for the pass band (or notch) width ΔF is known, then this is helpful, but not required.
2. (*Filter Design*) Place poles and zeros. First, find the digital center frequency, F_0.
   a. (Notch) Define zeros at $z_i = e^{\pm j2\pi F_0}$ and poles at $p_i = re^{\pm j2\pi F_0}$
   b. (Band pass) Define poles at $p_i = re^{\pm j2\pi F_0}$
   Pole radius may be found via $\Delta F \approx (1-r)/\pi$, if $r \approx 1$.
3. (*Filter Design*) Setup H(z) in the form $K_0 (1 - z_i\, z^{-1}) \;/\; (1 - p_i\, z^{-1})$. Where K_0 is a scaling factor, at your discretion. For pole radii near 1.0, try using $K_0 = (1 - r)$, for band pass and $K_0 = r$, for notch. These are reasonable estimates for a peak frequency response of ~1.0.
4. (*Filter Design*) Multiply factors in H(z) to identify A_k and B_k coefficients for the difference equation.
5. (*Filter Analysis*) Determine the frequency response and evaluate if it is satisfactory. If not, adjust poles and zeros in step 2, and repeat. The scaling factor in the numerator, K_0, may also be adjusted if the peak gain of the system is unsatisfactory.

A number of **guidelines** should be respected when locating poles and zeros:

- Place poles inside the unit circle, for stability.
- Use factors in the form described above, this yields a causal filter with minimum delay.
- When using complex poles and zeros, work with conjugate pairs. This provides real coefficients for your difference equation.