Useful Transforms and Relationships - DePiero, CalPoly State University

DTFT: Discrete-Time Fourier Transform

- $H(F) \Leftrightarrow h[n]$ where $H(F) = \sum_{n=-\infty}^{n=+\infty} h[n]\, e^{-j2\pi F n}$
- Principle Range: $-0.5 \le F \le 0.5$, F is a continuous variable for digital frequency
- $H(F)$ is the frequency response of a system with impulse response h(n)
- If x(n) = A cos(2 pi F_0 n) then y(n) = A $|H(F_0)|$ cos(2 pi F_0 n + $\langle H(F_0)$)
- Note that $H(0)$ is a sum, not a mean (there is a scaling by N with discrete FT's)
- Not computationally feasible

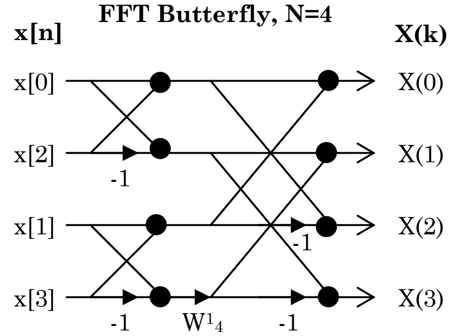
DFT: Discrete Fourier Transform

- $H(k) \underset{N}{\Leftrightarrow} h[n]$ where $H(k) = \sum_{n=0}^{n=N-1} h[n]\, e^{-j2\pi kn/N}$ for $0 \le k \le N-1$
- k is a sample index for digital frequency. N samples present in time and frequency domains.
- H(k) consists of samples of H(F): $H(k) = H(F_k)$ *for* $F_k = k/N$
- Computationally feasible

FFT: Fast Fourier Transform

- Described by a butterfly diagram, no formula.
- Variables k, N as with DFT. Also $W_N^k = e^{-j2\pi k/N}$
- In FFT $N = 2^r$, so signal may need to be zero padded.
- Computationally efficient!

FFT Butterfly, N=4



Z-Transform

- $H(z) \underset{z}{\Leftrightarrow} h[n]$ with $H(z) = \sum_{n=0}^{n=+\infty} h[n]\, z^{-n}$
- $H(z) = H(F)$ *for* $z = e^{j2\,pi\,F}$

Relationship Between $H(z)$ and Difference Equation for a 2nd Order System

- $y[n] = \sum_{k=0}^{k=M-1} B_k\; x[n-k] \;-\; \sum_{k=1}^{k=N-1} A_k\; y[n-k]$, M=N=3 for 2nd Order
- $H(z) = \dfrac{B_0 + B_1 z^{-1} + B_2 z^{-2}}{1 + A_1 z^{-1} + A_2 z^{-2}} = \dfrac{(1-\alpha_1\, z^{-1})}{(1-\rho_1\, z^{-1})}\dfrac{(1-\alpha_2\, z^{-1})}{(1-\rho_2\, z^{-1})}$, α are zeros and ρ are poles

Relationship between Frequency Axes and Unit Circle in Z Plane (S = sample rate in Hz)