EE302 Analog Controls – System Performance Measures - DePiero

This handout summarizes quantitative performance metrics. Both steady state and transient behavior is characterized.

Steady State Performance

Of primary concern for steady state operation is the steady state error, ess. This describes the error between a desired versus actual response. The discussion here is restricted to a unity gain feedback system, as shown below. Non-unity gain systems can be transformed into this simplified type.

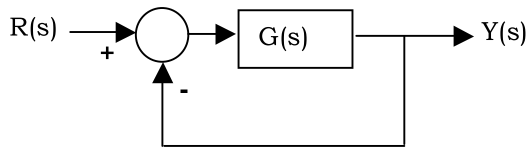

Table 1 describes steady state error for the unity gain feedback systems. Results vary depending on the type number of the system.

Type Number	Step Input	Ramp Input	Parabolic Input
0	A/(1+Kp)	Infinite	Infinite
1	0	A/Kv	Infinite
2	0	0	A/Ka

Table 1. Steady state error for various inputs and system types.

$K_p = \lim_{s\to 0} G(s)$ $K_v = \lim_{s\to 0} s\,G(s)$ $K_a = \lim_{s\to 0} s^2 G(s)$

Transient Performance

Transient performance (such as % overshoot) is often a concern when designing control systems. For a control engineer, the relationship between performance measures and closed loop pole locations is critical.

Underdamped second order systems are addressed herein. Recall that for 2nd order systems:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{and} \quad h(t) = \frac{\omega_n}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta\, t)$$

Where ω_n is the natural frequency, ζ is the damping ratio, and the parameter $\beta = \sqrt{1-\zeta^2}$. Figure 1 shows the corresponding pole locations.

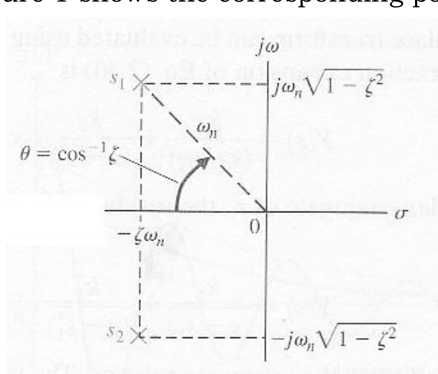

Figure 1. Pole/Zero plot for a 2nd order underdamped system. (From Dorf text.)

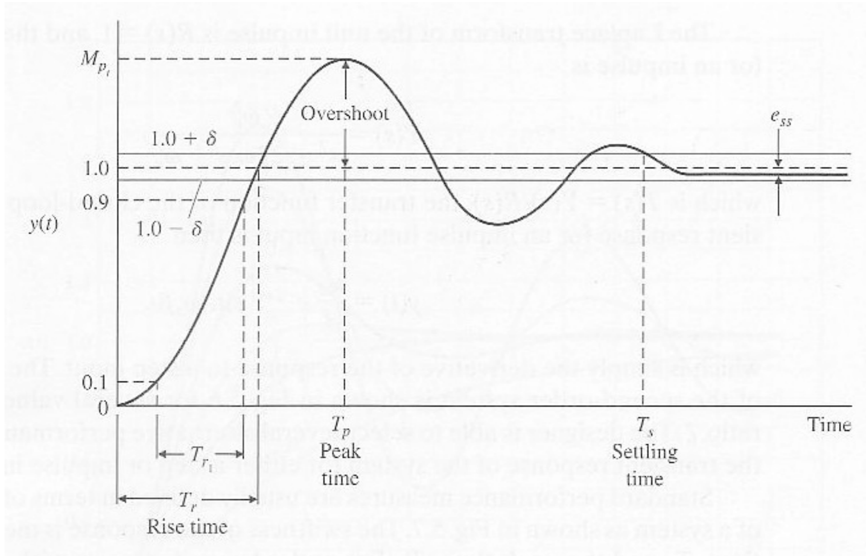

Figure 2. Metrics describing 2nd order response. (From Dorf text.)

Performance Measure	Approximation
Settling Time	$T_s = 4/(\zeta\ \omega_n)$
Peak Time	$T_p = \pi/(\omega_n\ \beta)$
Percent Overshoot	P.O. = 100 $e^{-\zeta\pi/\beta}$
Rise Time (10-90%)	$T_{r1} = (2.16\zeta + 0.6)/\omega_n$

Table 2. Approximations various performance measures.

The parameters ω_n and ζ may be estimated directly from experimental measurements. This is helpful for setting up a transfer function based on measured data. Given the damped frequency of oscillation observed for a step response, ω , and the number of visible cycles, n (until the response settles to within 2% of the final value). The damping ratio and natural frequency may then be estimated as:

$$\text{Damping Ratio} = \zeta = 0.55/n$$

$$\text{Natural Frequency} = \omega_n = \omega\ /\ \beta\ =\ \omega\ /\ \sqrt{1-\zeta^2}$$

The estimate of damping ratio is useful for $0.3 <= \zeta <= 0.8$.