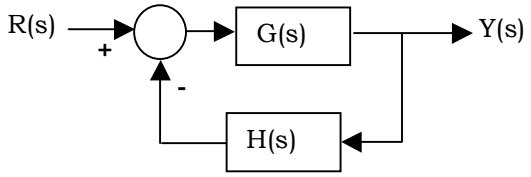
EE302 Analog Controls – Plotting a Root Locus - DePiero

This handout summarizes the steps needed to estimate a root locus plot for a system with T(s) = Y(s)/R(s), and with the forward and feedback blocks shown below. Several definitions precede the root locus procedure.



- Define GH(s) as the open loop transfer function. This assumes negative feedback. Also, note the (-) of the negated input to the summer is not included in GH(s) for root locus.
- Define Nz and Np as the number of zeros and poles, respectively, for GH(s).
- Define $\angle(s+z_i)$ as the angle of the vector difference between s and $-z_i$, a zero of GH(s).
- Define $\angle(s+p_j)$ as the angle of the vector difference between s and $-p_j$, a pole of GH(s).
- Define $\angle G(s)H(s) = \sum_{i=1}^{i=Nz} \angle(s+z_i) - \sum_{j=1}^{j=Np} \angle(s+p_j)$
- Define the 'Angle Criterion' as $\angle G(s)H(s) = 180^o$ *or equivalent* for a point $s = s_0$

1	Plot the poles & zeros of GH(s) in the s-plane	$GH(s) = \dfrac{\Pi_i(s+z_i)}{\Pi_j(s+p_j)}$
2	Plot portions of real axis to the left of an odd number of poles and zeros.	
3	The loci proceed from the poles to the zeros of GH(s). Zeros may be at infinity; in this case the loci proceed asymptotically. Find asymptote center σ_A and angles ϕ_A.	$\sigma_A = (\sum_j(-p_j) - \sum_i(-z_i))/(Np-Nz)$ $\phi_A = 180^o(2q+1)/(Np-Nz)$ $q = 0, 1, 2, \ldots Np-Nz-1$
4	Use Routh-Hurwitz criteria to find the gain K at which the locus crosses the imaginary axis.	
5	Find any breakaway (or break-in) points on the real axis.	Find $s=s_0$ where $\dfrac{d}{ds}\left\{\dfrac{-1}{G(s)H(s)}\right\} = 0$
6	Find the departure angles, γ_j, for poles. Use angle criterion at a point $s=s_0$, infinitesimally close to the pole p_j. Also find the arrival angles, β_i, associated with zeros.	Use the Angle Criterion: $\angle G(s)H(s) = 180^o$ *or equivalent*
7	Estimate the path of the root locus. • Np gives the #of sections of the locus • Locus is symmetric with respect to the real axis in the s-plane. • Each section of the locus is continuous, moving from poles to zeros of GH(s) • Locus curves don't generally cross	
8	Find values of K_x at any points of interest, $s=s_x$.	$K_x = 1/\lvert G(s_x)H(s_x)\rvert$