EE302 Analog Controls – Stability & Routh-Hurwitz Test - DePiero

This handout summarizes concepts of stability and stability testing. The bounded-input/bounded-output ('BIBO') criterion for stability defines a system as stable if, for all t, an input c(t) with $|c(t)|<M$ results in an output r(t) with $|r(t)|<P$, for finite M and P. It is a necessary condition for stability that all poles of the closed-loop transfer function, T(s), to reside in the left half of the s-plane. A system with roots on the jw axis is defined as marginally stable.

Before the advent of modern computers the Routh-Hurwitz Test was quite necessary. It reveals whether or not all the poles are in the left half plane, without expressly computing their values. To find roots with a tool such as MatLab, try 'help roots' to get started. A strength of Routh-Hurwitz that remains today is the ability to include system parameters (gain, K) in the analysis of stability. This permits critical values of these parameters to be ascertained – such as a gain that causes a system to go unstable.

Routh-Hurwitz Test

The version of the Routh-Hurwitz Test presented here does not allow for marginally stable systems (having roots on the jw axis). Also coverage here does not include pathological cases that can result, for example, due to perfectly symmetrical locations of poles in the LHP & RHP. (Perfect symmetry is unrealistic with real-world parameters).

The stability test involves an examination of entries in the Routh Array, which is defined based on the denominator polynomial of the closed-loop transfer function, T(s):

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_1 s + a_0$$

The associated Routh Array is then:

s^n		a_n	a_{n-2}	a_{n-4}	…
s^{n-1}		a_{n-1}	a_{n-3}	a_{n-5}	…
s^{n-2}		b_{n-1}	b_{n-3}	b_{n-5}	…
s^{n-3}		c_{n-1}	c_{n-3}	c_{n-5}	…
…		…	…	…	
s^0		h_{n-1}			

Where the a_i coefficients come from the original polynomial. The other coefficients are computed in a fashion similar to a determinant, by selecting appropriate values in the array. The pattern of values is illustrated via the following cases:

$$b_{n-1} = -\frac{1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} \quad \text{also} \quad b_{n-3} = -\frac{1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

The pattern continues in lower rows, such as:

$$c_{n-1} = -\frac{1}{b_{n-1}}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}$$

Some Notes

- Always work with values in the first column, while moving across a row.
- Blank entries at the ends of rows are treated as zero values.

The Routh-Hurwitz Test requires that there be no sign changes in the first column of the Routh Array, for the system to be stable. The number of sign changes is equal to the number of RHP poles.

This is best illustrated via examples…