EE525 - Notes on Kalman Filter - F. W. DePiero

Implementation, General Case

1) Update Kalman Gain matrix, $K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$
2) Acquire new measurement, z_k
3) Update State Estimate using new measurement, $\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-)$
4) Use State Estimate, as needed in application
5) Update Error Covariance matrix, $P_k = (I - K_k H_k) P_k^-$
6) Project ahead. Find state based on system dynamics, $\hat{x}_{k+1}^- = \phi_k \hat{x}_k$
7) Project ahead. Find Error Covariance based on dynamics & expected process noise, $P_{k+1}^- = \phi_k P_k \phi_k^T + Q_k$
8) Advance discrete-time index k = k + 1
9) Loop to step 1

Finding State Transition Matrix

Given system dynamics described by $\dot{x} = Fx$ (continuous-time) the corresponding discrete-time state equation is $x_{k+1} = \phi_k x_k + w_k$, and

$$\phi_k = L^{-1}[(sI - F)^{-1}], \quad t = \Delta t$$

Where $L^{-1}[]$ is the inverse Laplace Transform and Δt = sample period in seconds. The state vectors are Nx1 and the process noise for the discrete-time model, w_k, is a white noise sequence. (Also see below).

Finding Error Covariance Matrix

Given a process noise source, f(t), and given transfer functions $G_i(s) = X_i(s)/F(s)$ that relate f(t) to each of the components of the state vector x_i. The Error Covariance matrix $Q_k = E[w_k w_k^T]$ has (i,j) elements

$$E[x_i x_j] = \int_0^{\Delta t}\int_0^{\Delta t} g_i(\alpha)\, g_j(\beta)\, R_f(\alpha - \beta)\, d\alpha\, d\beta$$

Where $g_i(t)$ and $G_i(s)$ are Laplace Transform pairs, and $R_f()$ is the autocorrelation function for the random process f(t).

Specific Implementation

- Position Measurements
- State: Position, Velocity, Acceleration
- Gaussian White Noise (Infinite Variance), PSD = W
- Sample Period = Δt

$$\phi_k = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \qquad Q_k = \begin{bmatrix} \frac{W}{20}\Delta t^5 & \frac{W}{8}\Delta t^4 & \frac{W}{6}\Delta t^3 \\ \frac{W}{8}\Delta t^4 & \frac{W}{3}\Delta t^3 & \frac{W}{2}\Delta t^2 \\ \frac{W}{6}\Delta t^3 & \frac{W}{2}\Delta t^2 & W\Delta t \end{bmatrix}$$