EE 241 – Experiment #8: STEP RESPONSE OF A SECOND ORDER RLC CIRCUIT

PURPOSE:

- To observe the unit step response of a second-order series RLC circuit.
- To show that the natural response is either a damped oscillation or an exponential decay.
- To learn how to change the damping behavior of a 2nd order natural response (overdamped, underdamped, critically-damped) via adjusting the values of the circuit elements.
- To measure the essential features of a natural response directly from its captured display and compare them to the theoretical values.

This experiment relates to the following course learning objectives of the course:
1. Ability to interconnect equipment and devices such as multimeter, counters, and oscilloscopes to achieve required results.
2. Ability to relate practical laboratory results with lecture theory.
3. Ability to obtain and analyze time-domain response of second-order circuits.
4. Ability to analyze and evaluate data.

LAB EQUIPMENT:

1. Agilent 54621A Oscilloscope
2. Decade Resistance Box, 1Ω Step
1. Decade Capacitor, 0.01 μF Step
1. Decade Inductor, 10 mH Step
1. Agilent 34410A Digital Multimeter/Timer/Counter
1. Agilent 33120A Function Generator (FG)

STUDENT PROVIDED EQUIPMENT:

1. Meter lead set
2. Scope probes
4. Banana-to-banana leads
2. BNC-to-banana leads
2. BNC to BNC
1. BNC T

Experiment Sections:

1) Obtaining Step Response of 2nd order Series RLC Circuit
2) Time-Domain Measurements of 2nd order response

Section 1, Obtaining Step Response of Second-Order Series RLC Circuit

Note: To ensure a stable display, trigger the oscilloscope on the square wave input throughout this experiment.

a) Set up the circuit shown. Use resistor, inductor, and capacitor decade boxes for R, L, and C, respectively. Set L = 100 mH, and C = 0.01 μF. Using the multimeter, measure and record the parasitic resistance $R_p$ of the inductor (depending on the inductor, it should be in the range of 4 -15Ω). $R_{th}$ represents the finite output.

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1 Version 6, last revised 3/1/12, EE Dept., Cal Poly
resistance of the function generator (Use the value measured in Lab 4, Section 1, Step d — it should be close to 50Ω). Place the FG output termination to “HIGH Z” and set it to produce a 2V peak to peak square wave with a period of 10 ms (frequency of 100 Hz) and 1V DC offset. Note that the resulting amplitude of the (shifted) square wave rises from 0 to 2 volts.

\[ \text{Percent Overshoot} = 100 \times \frac{V_{\text{max}} - V_0(\infty)}{V_0(\infty)} \]

b) Set the resistor decade box to \( R = 1k\Omega \) and monitor the input and output voltage waveforms of the circuit using the channels one and two of the oscilloscope, respectively. Observe that the circuit is underdamped. Adjust the scope settings as appropriate to expand and clearly reveal the oscillation (ringing). Capture the scope’s display and include a copy of it in your report.

c) Set the resistor decade box to \( R = 7k\Omega \). The circuit should now be overdamped. Adjust the scope settings appropriately to reveal the overdamped natural response. Include the response in the lab report.

d) Change the value of the resistor box \( R \) to obtain the resistance \( R_x \) at which the response is critically-damped. Measure and record the actual value \( R_x \) using MM. Your response will be critically-damped when a slight change in \( R \) produces ringing in the response. Record the value of \( R \) that produces the critically-damped response and copy the response in the lab report. Also set \( R \) to pre-lab calculated value and capture the image.

**Question: Section 1**

1) In step (d) above, what is the percent difference between the value of \( R'_c = R_x + R_{TH} + R_p \) and the calculated value \( R_c \) obtained in the prelab? Justify the difference.

**Section 2) Time-Domain Measurements of the Characteristics of Second-Order Response**

*Note: In the following steps, features of the natural response will be measured by direct measurements from displays on the scope. Use “cursors” utility of the scope to make these measurements. To make accurate measurements, it is recommended to enlarge the region of the waveform needed for the measurement. In addition, it is best to lower the intensity of the display to make the plot thinner and hence easier to read intersections with cursor lines.*

a) For the under-damped response, measure the step response overshoot using the equation:
Note: The underdamped response shown below corresponds to an input square wave whose (shifted) amplitude rises from 0 to \( V_o(\infty) \) volts.

Use scope’s “overshoot” softkey (accessible in “quick meas” mode) to read the percent overshoot and compare it with the value obtained using the above formula. When using the scope cursors, be sure to measure \( V_{\text{max}} \) and \( V_o(\infty) \) relative to the baseline of the response (i.e., the value at time 0).

b) Determine the frequency of oscillation (damped frequency \( \omega_d \)) by measuring the period of oscillation as shown in the figure above. Note that once the scope cursors are set to read the period \( \Delta t \), it will also display the frequency \( f = 1/\Delta t \).

c) Read this entire part before making measurements. One way to verify that the output responses obtained in the previous section represent the solution to the second-order differential equations that describe them is to compare the measured time constants of the output responses to their calculated values. For the overdamped and critically-damped cases, direct measurement of the time constant from the response is feasible only under certain conditions (see questions 3 and 4 in this section). Here, we will only measure the time constant for the underdamped case.

The time constant of an exponential decay of the form \( A e^{-t/\tau} \) is the time required for the decay to reach \( 1/e \) of its maximum (i.e., \( A/e \)). As shown in the diagram above, for the underdamped case the envelope of the oscillation is an exponential decay that can be modeled as:

\[
Env(t) = V_o(\infty) + [V_o(0) - V_o(\infty)] e^{-t/\tau}
\]

where
- \( Env(t) \): Envelope of oscillation for underdamped case
- \( V_o(0) \): Initial value (a constant)
- \( V_o(\infty) = Env(\infty) \): Final value
- \( \tau \): Time constant
The time constant $\tau$ can be obtained via time-voltage measurements at two points on the envelope function $Env(t)$. The two points $[t_1, V_o(t_1)]$ and $[t_2, V_o(t_2)]$ can be any two points on the envelope function. Assuming $t_2 > t_1$, the time constant $\tau$ can be calculated from the following formula shown on the right.

$$\tau = \frac{t_2 - t_1}{\ln \left( \frac{V_o(t_1) - V_o(\infty)}{V_o(t_2) - V_o(\infty)} \right)}$$

Note that it is advisable to choose the two points $(t_1, v_1)$ and $(t_2, v_2)$ far from each other in order to increase the accuracy of measurement of $\tau$. For this case, it is advisable to select $t_1$ and $t_2$ to correspond to first and third peaks of oscillation.

**Questions: Section 2**

1) What is the percent difference between the measured value of the damping frequency $\omega_d$ in step (b) above and the calculated value obtained in the prelab? Explain reasons for the difference.

2) Record the calculated and measured value of the time constant for the underdamped case. Compute the percent difference between the two and explain reasons for any difference.

3) Derive the equation for time constant $\tau$ in step (c) above. Hint: Substitute each of the two data points $[t_1, V_o(t_1)]$ and $[t_1, V_o(t_2)]$ into the envelope function $Env(t) = V_o(\infty) + [A - V_o(\infty)] e^{-t/\tau}$ to form two equations. Divide one equation by the other (side by side), simplify, take the natural log of the result, and solve for $\tau$. Note that $Env(t_1) = V_o(t_1)$ and $Env(t_2) = V_o(t_2)$.

4) As read in the narrative section of this experiment, the overdamped response is actually the sum of two exponentials having different time constants $\tau_1$ and $\tau_2$, i.e.,

$$V_o(t) = k_1 e^{-t/\tau_1} + k_2 e^{-t/\tau_2} + V_o(\infty)$$

Assuming that $\tau_2$ is greater than $\tau_1$ (that is, the $e^{-t/\tau_1}$ term approaches zero sooner than $e^{-t/\tau_2}$), prove that we can still use the equation in part (c) above to calculate the larger of the two time constants $\tau_1$ and $\tau_2$. Hint: use the hint in question 3 to derive the equation for $\tau_2$. 

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