Bipolar Junction Transistor Transport Model

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This supplement presents the bipolar junction transistor Transport model, explains how it predicts DC BJT behavior, and shows how to simplify the model for four common modes of BJT operation: forward active, saturation, reverse active, and cut-off.

Index Terms—Bipolar transistors, bipolar integrated circuits, integrated circuit modeling, semiconductor device modeling, semiconductor devices.

I. INTRODUCTION AND LEARNING OBJECTIVES

The Ebers-Moll[1] and Gummel-Poon[2] models predict detailed large- and small-signal BJT behavior to support hand analysis or computer simulations. The Transport model offers a slightly simpler approach while preserving intuition and accuracy. The Transport model circuit topology derives deductively from the Ebers-Moll model via circuit theory. Alternately, superposing forward active and reverse active mode equivalent circuits permits assembling the Transport model equivalent circuit inductively [3]. The latter strategy, conveniently proceeds from the EE 306 text treatment [4].

This supplement informs the following learning objectives:
(a) an ability to draw the Transport model equivalent circuit
(b) an ability to use the Transport model to predict BJT terminal voltages and currents
(c) an ability to draw simplified Transport model equivalent circuits for the forward active, saturation, reverse active, and cut-off operation modes.

Table I outlines the supplement organization. Section II presents the Transport model equivalent circuits and circuit equations for the NPN and PNP model versions. Section III simplifies the Transport model for the four BJT operation modes. An appendix describes the Ebers-Moll model.

II. ASSEMBLING THE TRANSPORT MODEL

A. NPN BJT

We begin with an overview. Equations 1 relate the BJT terminal currents ($i_C$, $i_E$, and $i_B$) to the Base-Emitter ($V_{BE}$) and Base-Collector ($V_{BC}$) voltage drops at a given thermal voltage ($V_T$). The saturation current ($I_S$), the forward current gain ($\beta_I$), and the reverse current gain ($\beta_R$) describe a given transistor. Figure 1 contains the Transport model equivalent circuit for an NPN-BJT. Equations 2 define the transport current ($i_T$), forward current ($i_F$), and reverse current ($i_R$).

$$i_C = I_S [\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)] - \frac{I_S}{\beta_R} [\exp\left(\frac{V_{BC}}{V_T}\right) - 1]$$  \hspace{1cm} (1C)

$$i_E = I_S [\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)] + \frac{I_S}{\beta_F} [\exp\left(\frac{V_{BE}}{V_T}\right) - 1]$$  \hspace{1cm} (1E)

$$i_B = \frac{I_S}{\beta_F} [\exp\left(\frac{V_{BE}}{V_T}\right) - 1] + \frac{I_S}{\beta_R} [\exp\left(\frac{V_{BC}}{V_T}\right) - 1]$$  \hspace{1cm} (1B)

Table I

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Fig. 1. NPN–BJT Transport model equivalent circuit juxtaposed on the NPN–BJT cross section. Per convention, positive base current ($i_B$) and collector current ($i_C$) flow into the device. Positive emitter current ($i_E$) leaves the device.
The NPN transistor formulation assumes positive currents flow into the device. For the NPN BJT, positive base current (\(i_b\)) and collector current (\(i_C\)) flows into the device. For the PNP BJT, positive emitter current (\(i_E\)) and collector current (\(i_C\)) leave the device.


A. Forward Active Mode

In Forward Active Mode (FA), the Base-emitter junction (BE junction) turns on, and the Base-collector junction (BC junction) turns off. The large BE junction exponential terms greatly exceed the negligible BC junction terms and 1 terms, so Equations 1 simplify as follows for FA:

\[
\begin{align*}
    i_C &= i_S \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) \right] - \frac{i_F}{\beta_R} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] \\
    &= i_S \left[ \exp \left( \frac{V_{BE}}{V_T} \right) \right] (\beta_F + 1) \\
    \end{align*}
\]

(5C)

\[
\begin{align*}
    i_E &= i_S \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) \right] + \frac{i_F}{\beta_R} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] \\
    &= i_S \left[ \exp \left( \frac{V_{BE}}{V_T} \right) \right] (\beta_F + 1) \\
    \end{align*}
\]

(5E)

\[
\begin{align*}
    i_B &= \frac{i_S}{\beta_R} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] \\
    &= \frac{i_S}{\beta_R} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) \right] \\
    \end{align*}
\]

(5B)

Equations 5 support the equivalent circuit simplification shown in Figure 4. As required, Kirchhoff’s Current Law applies in FA at the emitter terminal:

\[
i_E = i_B + i_C = (\beta_F + 1)i_B
\]

(6)

B. Reverse Active Mode

RA operation occurs less frequently in practice than the other operation modes, primarily due to the low reverse current gain (\( \beta_R \)) compared to forward current gain (\( \beta_F \)). \( \beta_R \) usually has a value less than 10 and often less than 1, whereas \( \beta_F \) usually exceeds 20 and often exceeds 100.

In Reverse Active Mode (RA), the Base-emitter junction (BE junction) turns off, and the Base-collector junction (BC junction) turns on. The large BC junction exponential terms greatly exceed the negligible BE junction terms and -1 terms, so Equations 1 simplify into Equations 7 as follows:

\[
\begin{align*}
    i_C &= i_S \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - \exp \left( \frac{V_{BE}}{V_T} \right) \right] - \frac{i_F}{\beta_R} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \\
    &= -i_S \left[ \exp \left( \frac{V_{BC}}{V_T} \right) \right] (\beta_R + 1) \\
    \end{align*}
\]

(7C)

\[
\begin{align*}
    i_E &= i_S \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - \exp \left( \frac{V_{BE}}{V_T} \right) \right] + \frac{i_F}{\beta_R} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \\
    &= -i_S \left[ \exp \left( \frac{V_{BC}}{V_T} \right) \right] \\
    \end{align*}
\]

(7E)

\[
\begin{align*}
    i_B &= \frac{i_S}{\beta_R} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \\
    &= \frac{i_S}{\beta_R} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) \right] \\
    \end{align*}
\]

(7B)

Because the negative signs on the collector and emitter terminals can make circuit analysis more confusing than necessary, it often proves more convenient to redefine the directions of positive collector current (\( i_C \)) and positive emitter current (\( i_E \)). Therefore, for reverse active mode, we define positive emitter current flowing into the emitter terminal and positive collector current flowing out the collector terminal. This convention, shown in Figure 5, does not agree with the convention used for forward active mode shown in Figure 4.

Rewriting Equations 7 using the new convention yields Equations 8 for RA.

\[
\begin{align*}
    i_C &= i_S \left[ \exp \left( \frac{V_{BC}}{V_T} \right) \right] (\beta_R + 1) \\
    i_E &= i_S \left[ \exp \left( \frac{V_{BC}}{V_T} \right) \right] \\
    i_B &= \frac{i_S}{\beta_R} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) \right] \\
    \end{align*}
\]

(8C)

(8E)

(8B)

Equations 8 support the equivalent circuit simplification shown in Figure 5. As required, Kirchhoff’s Current Law
applies in RA at the collector terminal:

\[ i_c = i_B + i_E = (\beta_R + 1)i_B \]  \hspace{1cm} (9)

C. Cut-off Mode

In Cut-off Mode (CO), both junctions turn off. With both junctions reversed biased, the small exponential terms become negligible compared to the \(-1\) terms, so Equations 1 simplify as follows for CO, leaving only leakage currents:

\[ i_c = i_s \exp \left( \frac{V_{RE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) - \frac{i_s}{\beta_R} \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \]  \hspace{1cm} (10C)

\[ i_E = i_s \exp \left( \frac{V_{RE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) + \frac{i_s}{\beta_P} \exp \left( \frac{V_{RE}}{V_T} \right) - 1 \]  \hspace{1cm} (10E)

\[ i_B = \frac{i_s}{\beta_P} \exp \left( \frac{V_{RE}}{V_T} \right) - 1 \]  \hspace{1cm} (10B)

Equations 10 support the equivalent circuit simplification shown in the left pane of Figure 6. Typically, hand calculations ignore the negligible leakage currents and use the following approximation:

\[ i_B \approx i_c \approx i_E \approx 0 \]  \hspace{1cm} (11)

In CO, the BJT behaves as an open circuit. Figure 6 shows the Equation 11 open circuit approximation in its right pane.

D. Saturation Mode

In Saturation Mode (SAT), both junctions turn on, so the \(-1\) terms in Equations 1 become negligible compared to the exponential terms. Equations 1 simplify as follows for SAT:

\[ i_c = i_s \exp \left( \frac{V_{RE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) - \frac{i_s}{\beta_R} \exp \left( \frac{V_{BC}}{V_T} \right) \] \hspace{1cm} (12C)

\[ i_E = i_s \exp \left( \frac{V_{RE}}{V_T} \right) - \exp \left( \frac{V_{BC}}{V_T} \right) + \frac{i_s}{\beta_P} \exp \left( \frac{V_{RE}}{V_T} \right) \] \hspace{1cm} (12E)

\[ i_B = \frac{i_s}{\beta_P} \exp \left( \frac{V_{RE}}{V_T} \right) + \frac{i_s}{\beta_R} \exp \left( \frac{V_{BC}}{V_T} \right) \] \hspace{1cm} (12B)

In Saturation Mode, \( \beta_P i_B \geq i_c \), so hand analysis proceeds more conveniently by considering terminal voltages. Combining Equations 12C and 12B produces

\[ V_{BE(SAT)} = V_T \ln \left( \frac{\beta_R (1 + \beta_R) i_B + i_c}{i_s} \right) \] \hspace{1cm} (13)

and

\[ V_{BC(SAT)} = V_T \ln \left( \frac{\beta_P i_B i_c}{i_s} \right) \] \hspace{1cm} (14)

\[ V_{CE(SAT)} = V_{BE(SAT)} - V_{BC(SAT)} \] \hspace{1cm} (15)

Inserting Equations 13 and 14 into Equation 15 produces

\[ V_{CE(SAT)} = V_T \ln \left( \frac{\beta_P (1 + \beta_R)}{\beta_R + \beta_P + 1} \right) \] \hspace{1cm} (16)

The preceding formulations use the common-emitter current gain, \( \beta \). Equivalent versions based on the common-base current gain, \( \alpha \), also exist. To translate from one to the other, use

\[ \alpha = \frac{\beta}{\beta + 1} \] \hspace{1cm} (17)

or

\[ \beta = \frac{\alpha}{1 - \alpha} \] \hspace{1cm} (18)
E. Edge of Conduction and Edge of Saturation

Figure 8 shows equivalent circuits for the Edge of Conduction (EOC) and Edge of Saturation (EOS) modes. **EOC** describes operation at the corner case between CO and FA modes as the BJT almost turns on or just turns off. At **EOC**, the BE junction drops almost enough voltage to cause non-negligible base current, but neither junction turns on or conducts significant current. In Figure 8A, the voltage, $V_{BE(EOC)}$, recognizes the BE junction voltage drop, and the open circuit recognizes the lack of current.

**EOS** describes operation at the corner case between FA and SAT modes as the BJT operates with the collector and base currents obeying the FA relation $I_C = \beta_F I_B$, and the terminal voltages reach their saturation values. In Figure 8B, the voltage drops $V_{BE(EOS)} = V_{BE(SAT)}$ and $V_{CE(EOS)} = V_{CE(SAT)}$ capture the saturation behavior.

Table IV places **EOC** on the boundary between CO and FA, while **EOS** sits on the boundary between FA and SAT. Table V shows typical values for Silicon BJT parameters applied to the equivalent circuits derived from the Transport Model [5].

**TABLE IV**

<table>
<thead>
<tr>
<th>BC Junction Off</th>
<th>BC Junction On</th>
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</thead>
<tbody>
<tr>
<td><strong>BE Junction Off</strong></td>
<td><strong>Forward Active</strong></td>
</tr>
<tr>
<td>$I_C \approx I_E \approx I_B \approx 0$</td>
<td>$I_C = \beta_F I_B$</td>
</tr>
<tr>
<td><strong>EOC</strong></td>
<td><strong>Saturated</strong></td>
</tr>
<tr>
<td><strong>EOS</strong></td>
<td>$I_C &lt; \beta_F I_B$</td>
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**TABLE V**

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<tr>
<th><strong>Typical Silicon BJT Parameters for Hand Calculations [5]</strong></th>
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<tr>
<td>$V_{BE(EOC)}$</td>
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<tr>
<td>$V_{BE(ON)}$</td>
</tr>
<tr>
<td>$V_{BE(SAT)} = V_{BE(EOS)}$</td>
</tr>
<tr>
<td>$V_{CE(SAT)} = V_{CE(EOS)}$</td>
</tr>
<tr>
<td>$V_{BC(ON)} = V_{BC(SAT)}$</td>
</tr>
</tbody>
</table>

Fig. 8. **EOC** and **EOS** equivalent circuits. A) At **EOC**, the BE junction almost has a large enough voltage drop to turn on, but neither junction conducts significant current. The voltage, $V_{BE(EOC)}$, recognizes the BE junction voltage drop, and the open circuit recognizes the lack of current. B) At **EOS**, the transistor just leaves Forward Active Mode and enters Saturation. FA preserves the current relation $I_C = \beta_F I_B$, and SAT accounts for the $V_{BE(EOS)}$ and $V_{CE(EOS)}$ voltage drops.
Because the Transport model derives from the Ebers-Moll model, this appendix presents the Ebers-Moll model following the Hodges and Jackson treatment [6]. Figure A1 shows the model in terms of the diode currents, $i_{DE}$ and $i_{DC}$ shown in Equations A1:

\[ i_{DE} = I_{ES} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] \]  
(A1E)

\[ i_{DC} = I_{CS} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \]  
(A1C)

Applying Kirchhoff’s Current Law at the collector and emitter terminals in Figure A1 gives Equations A2:

\[ i_E = i_{DE} - \alpha_R i_{DC} \]  
(A2E)

\[ i_C = \alpha_F i_{DE} - i_{DC} \]  
(A2C)

Inserting Equations A1 into Equations A2 gives the general Ebers-Moll Equations:

\[ i_E = I_{ES} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] - \alpha_R I_{CS} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \]  
(A3E)

\[ i_C = \alpha_F I_{ES} \left[ \exp \left( \frac{V_{BE}}{V_T} \right) - 1 \right] - I_{CS} \left[ \exp \left( \frac{V_{BC}}{V_T} \right) - 1 \right] \]  
(A3C)

\[ i_B = i_E - i_C \]  
(A3B)

Note that the parameters $I_{ES}, I_{ES}, \alpha_F$, and $\alpha_R$ describe BJT behavior for the Ebers-Moll model.

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This supplement uses LTspice IV to create the circuit diagrams [7].

REFERENCES


David Braun (M’97–SM’03) received the B.S. and M.S. degrees in electrical engineering from Stanford University, Stanford, CA, in 1985 and 1986 and the Ph.D. degree in electrical and computer engineering from the University of California at Santa Barbara in 1991.

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Prof. Braun is a member of the American Society for Engineering Education and the American Physical Society. He received the IEEE Third Millennium Medal from the IEEE Central Coast Section.

Fig. A1. NPN–BJT Ebers-Moll model equivalent circuit. Per convention, positive base current ($i_B$) and collector current ($i_C$) flow into the device. Positive emitter current ($i_E$) leaves the device. The equivalent circuit corresponds to Equations A3.