Truncated Multiplication for Image Compression

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Agenda

- Hardware multipliers
- Introduction JPEG image compressor
- Truncated multipliers in JPEG
- Results & Conclusions
- Q&A

2’s complement multiplication

- Takes two arguments of length *N*
- Produces *N* partial products
- Maximum output of length 2 *N*
- Result calculated by summing partial products

$$P = a_{n-1}b_{n-1}2^{2n-2} + \sum_{i=0}^{n-2}\sum_{j=0}^{n-2} a_i b_j 2^{i+j} + \sum_{i=0}^{n-2}(\overline{b_{n-1}a_i} + \overline{a_{n-1}b_i})2^{i+n-1} + 2^{2n-1} + 2^n$$

Example of 6 by 6 multiplier

column	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
							a_5	a_4	a_3	a_2	a_1	a_0
						X	b_5	b_4	b_3	b_2	b_1	b_0
						1	$(a_5b_0)'$	a_4b_0	a_3b_0	a_2b_0	a_1b_0	a_0b_0
						$(a_5b_1)'$	a_4b_1	a_3b_1	a_2b_1	a_1b_1	a_0b_1	
					$(a_5b_2)'$	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2		
				$(a_5b_3)'$	a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3			
			$(a_5b_4)'$	a_4b_4	a_3b_4	a_2b_4	a_1b_4	a_0b_4				
+	1	a_5b_5	$(a_4b_5)'$	$(a_3b_5)'$	$(a_2b_5)'$	$(a_1b_5)'$	$(a_0b_5)'$					
	P_{11}	P_{10}	P_9	P_8	P_7	P_6	P_5	P_4	P_3	P_2	P_1	P_0

Problems with multiplication

- Complexity
- Delay
- Word growth

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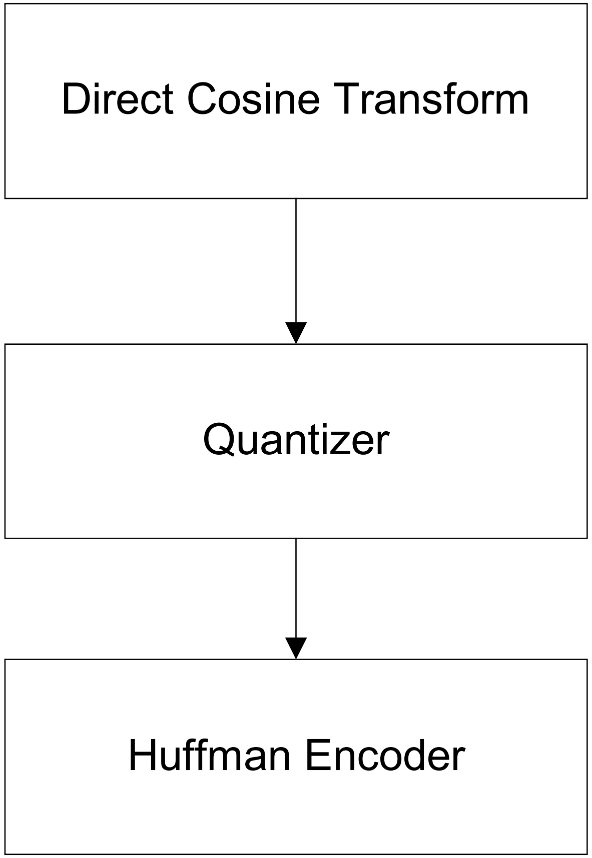
JPEG image compression

- The image is broken down into components, i.e. rgb
- Each component is broken down into blocks that are 8 pixels by 8 pixels.
- Each block is encoded separately

JPEG image compression (cont.)



Direct cosine transform stage

- This stage of the encoder is loss-less
- Output of the DCT is real
- For every input into DCT there is an output

$$F(u,v) = \frac{1}{4} C(u) C(v) \left[\sum_{x=0}^{\tau} \sum_{y=0}^{\tau} f(x,y) \bullet \cos \frac{(2x+1)u\pi}{16} \bullet \cos \frac{(2y+1)v\pi}{16} \right]$$

$$C(z) = 1\sqrt{2} \text{ , for } z = 0$$

$$C(z) = 1 \text{ , otherwise}$$

Quantization stage

- Each entry in the block is divided by a quantization value
- The quantization table can be any group of values, but is constant for a given component
- The most important results of the DCT tend to be in the upper right hand corner of any given block

Standard luminance table

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

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Observations about JPEG

- Hardware DCT accelerators can be quite large because of the number of multiplications
- Because the quantizer will remove the least significant bits why bother calculating them?
- Trend is towards larger images

DCT algorithm

- Implements 2-D DCT by doing a 1-D DCT on each row and a 1-DCT on the columns
- Uses 12 multiplications and 32 adds for each for each row or column
- Total of 768 multiplications and 2048 adds for each block

DCT algorithm (cont.)

- Total word growth is from 8 to 16 bits for the two stages
- Quantization values are shifted by 8 bits to remove the effects of word growth

6 by 6 multiplier with known word growth

column	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
							a_5	a_4	a_3	a_2	a_1	a_0
						X	b_5	b_4	b_3	b_2	b_1	b_0
						1	$(a_5b_0)'$	a_4b_0	a_3b_0	a_2b_0	a_1b_0	a_0b_0
						$(a_5b_1)'$	a_4b_1	a_3b_1	a_2b_1	a_1b_1	a_0b_1	
					$(a_5b_2)'$	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2		
				$(a_5b_3)'$	a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3			
			$(a_5b_4)'$	a_4b_4	a_3b_4	a_2b_4	a_1b_4	a_0b_4				
+	1	a_5b_5	$(a_4b_5)'$	$(a_3b_5)'$	$(a_2b_5)'$	$(a_1b_5)'$	$(a_0b_5)'$					
	P_{11}	P_{10}	P_9	P_8	P_7	P_6	P_5	P_4	P_3	P_2	P_1	P_0

Multiplier Truncation

- By reducing the number of columns calculated we can reduce the area, delay, and power for a hardware multiplier
- Reduced word growth
- Error introduced by not calculating carries for partial products

Truncated Multipliers

- Each multiplier (M) is described by the input width (a), the output width (p), and the number of ignored columns (i).
- Each row contains the following multipliers

Multiplier	Number in design
M[8,11,3]	7
M[8,11,8]	3
M[8,11,10]	2

6 by 6 multiplier with 3 bit truncation

column	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
							a_5	a_4	a_3	a_2	a_1	a_0
						X	b_5	b_4	b_3	b_2	b_1	b_0
						1	$(a_5b_0)'$	a_4b_0	a_3b_0	a_2b_0	a_1b_0	a_0b_0
						$(a_5b_1)'$	a_4b_1	a_3b_1	a_2b_1	a_1b_1	a_0b_1	
					$(a_5b_2)'$	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2		
				$(a_5b_3)'$	a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3			
			$(a_5b_4)'$	a_4b_4	a_3b_4	a_2b_4	a_1b_4	a_0b_4				
+	1	a_5b_5	$(a_4b_5)'$	$(a_3b_5)'$	$(a_2b_5)'$	$(a_1b_5)'$	$(a_0b_5)'$					
	P_{11}	P_{10}	P_9	P_8	P_7	P_6	P_5	P_4	P_3	P_2	P_1	P_0

Truncated Multipliers (cont.)

- Four of the columns use the following multipliers

Multiplier	Number in design
M[11,16,6]	7
M[11,16,8]	4
M[11,16,11]	1

Truncated Multipliers (cont.)

- The remaining four of the columns use the following multipliers

Multiplier	Number in design
M[11,16,7]	7
M[11,16,11]	4
M[11,16,13]	1

- The space savings is ~41%

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Uncompressed image

Standard compression

Truncated compression

Side by side comparison



Areas for further improvement

- Reduced height partial product array
- Combining the DCT stage with quantizaion

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