In this project you will implement a 2-D Fast-Fourier Transform (FFT), given a 1-D version. While the FFT has many uses for filtering and image enhancement, this project focuses on object recognition. The magnitude of the FFT has the benefit of translational invariance, thus eliminating the search associated with correlation-based methods of object recognition. In this project you will examine the distribution of energy in the frequency plane and will determine your own recognition criteria to distinguish between two images. The energy distribution $E(r)$ will be computed as a 1-D function of the radial distance, $r$, to DC in the frequency plane, thus providing rotational invariance.

**Project Requirements:**
Write a program that computes the 2D FFT of an image. Process two input images with your program and compute the energy distributions $E_1(r)$ and $E_2(r)$ for each. Examine plots of $E_1(r)$ and $E_2(r)$ and determine a value $r=r_0$, useful for distinguishing the two images. Compute the margin percentage: $100\% \times \frac{|E_1(r_0) - E_2(r_0)|}{\text{Max}(E_1(r_0), E_2(r_0))}$.

- Compute a 2-D FFT using the 1-D FFT provided with the development package.
- The frequency plane should be centered, and magnitude used for $E(r)$.
- Use either linear or log-based scaling method for $E(r)$, your choice.
- You may use square images, with side lengths that are a power of 2.
- Use images that each contain a single (large) letter: your first and last initials.

**Report**
1) A listing of your program, with comments, and file header giving your name, course and project number.
2) Brief report including:
   a) Your name, the course title.
   b) Example images.
   c) Plot of $E(r)$ for each input image.
   d) Brief discussion of procedure and comments on results.

**Questions**
1) The following image was digitized with 3 bit gray levels. Find the optimal Huffman codebook for the image. Find the expected compression, $C$, for the 4x4 image.

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2) Careful examination of some blobs reveals that their gray levels have a Normal distribution, with a mean of 200 and a standard deviation of 20. Find upper and lower thresholds, $t_1$ and $t_2$, for grayscale values so that a 0.85 probability exists that gray level values between $t_1$ and $t_2$ are part of a blob (these are inliers). Hint: use a table for the Normal distribution. Also, describe a way to compute $t_1$ and $t_2$ in a program (not using a table).

3) When working with binary values (1's and 0's only) a more efficient version of the median sort is possible that does not involve sorting. The better version requires only counting occurrences of binary values. Describe the algorithm.

4) How many complex multiplications are needed to transform an $N \times N$ image, given that a 1-D FFT requires $N \log_2 N$?