DSP Notes - Some Useful Transforms and Relationships

DTFT: Discrete-Time Fourier Transform
- $H(w) \Leftrightarrow h(n)$ where $H(w) = \sum_{n=-\infty}^{\infty} h(n) e^{-jwn}$
- Fundamental Interval: $-\pi \leq w \leq \pi$, $w$ is a continuous frequency variable (not sampled)
- $H(w)$ is the frequency response of a system with impulse response $h(n)$.
- If $x(n) = A \cos(w_0 n)$ then $y(n) = A \left| H(w_0) \right| \cos( w_0 n + \theta(w_0) )$
- Note that $H(0)$ is a sum, not a mean (there is a scaling by $N$ in discrete Fourier transforms)
- Not computationally feasible

DFT: Discrete Fourier Transform
- $H(k) \Leftrightarrow h(n)$ where $H(k) = \sum_{n=0}^{N-1} h(n) e^{-j\frac{2\pi kn}{N}}$ for $0 \leq k \leq N - 1$
- $k$ is a discrete frequency variable. $N$ samples in both the time and frequency domains.
- $H(k)$ consists of samples of $H(w)$: $H(k) = H(w_k)$ for $w_k = \frac{2\pi k}{N}$
- Computationally feasible

FFT: Fast Fourier Transform
- Described by a butterfly diagram, no formula.
- Variables $k, N$ as with DFT. Also $W_N^k = e^{-j\frac{2\pi k}{N}}$
- In FFT $N = 2^r$, so signal may need to be zero padded.
- Computationally efficient!

Z-Transform
- $H(z) \Leftrightarrow h(n)$ with $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$
- $H(z) = H(w)$ for $z = e^{jw}$

Relationship Between $H(z)$ and Difference Equation for a 2nd Order System
- $y(n) = \sum_{k=0}^{k=M-1} b_k x(n-k) - \sum_{k=1}^{k=N-1} a_k y(n-k)$, $M=N=3$ for 2nd Order
- $H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{(1 - \alpha_1 z^{-1}) (1 - \alpha_2 z^{-1})}{(1 - \rho_1 z^{-1}) (1 - \rho_2 z^{-1})}$, $\alpha$ are zeros and $\rho$ are poles

Relationship between Frequency Axes and Unit Circle in Z Plane

$F (Hz)$
$-Fs/2$ $0$ $Fs/2$ $Fs$
$w (rads/sample)$
$-\pi$ $0$ $\pi$ $2\pi$
$k$
$0$ $N/2$ $N$

$z = -1, w = \pi$
$z = j, w = -\pi / 2$
$z = 1, w = 0$