This method uses a least-squares formulation to fit $N$ planar data points $(x_i, y_i)$ to a circle. The error in radius is minimized (as opposed to the error in a single coordinate – as with line fitting via linear regression). An assumption is made in this version that the input data points arrive in pairs that have the same $x$ coordinate. A constant separation between the data points (in the $x$ direction) is not assumed. The technique computes center, $(x_c, y_c)$, and radius, $r$, of the circle.

The $y$ coordinate of the center of the circle, $y_c$, may be computed directly, simply by averaging the $N$ given $y_i$ data points.

$$y_c = \frac{1}{N} \sum_{i} y_i$$

The $y$ coordinates may then be re-expressed with respect to $y_c$, by defining

$$\tilde{y}_i = y_i - y_c$$

If the raw data points $(x_i, y_i)$ fit the circle reasonably well, then

$$(x_i - x_c)^2 + \tilde{y}_i^2 \approx r^2$$

expanding this

$$x_i^2 - 2x_c x_i + x_c^2 + \tilde{y}_i^2 \approx r^2$$

and rearranging to help identify unknowns gives

$$(-2x_i)(x_c) + (x_c^2 - r^2) \approx -\tilde{y}_i^2 - x_i^2$$

which permits a vector product to be written as

$$\begin{bmatrix} -2x_i & 1 \\ x_c & x_c^2 - r^2 \end{bmatrix} \approx \begin{bmatrix} -\tilde{y}_i^2 - x_i^2 \end{bmatrix}$$
This vector product may be used to setup each row of a matrix equation $A z = b$ as in

\[
\begin{bmatrix}
-2x_1 & 1 \\
-2x_2 & 1 \\
\vdots & \vdots \\
-2x_N & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_c \\
x_c^2 - r^2 \\
\vdots \\
x_c^2 - r^2 \\
\end{bmatrix}
\approx
\begin{bmatrix}
-\tilde{y}_1^2 - x_1^2 \\
-\tilde{y}_2^2 - x_2^2 \\
\vdots \\
-\tilde{y}_N^2 - x_N^2 \\
\end{bmatrix}
\]

which may be solved by least-squares.

A shortcoming of this approach is that the radius, $r$, is not an explicit unknown. (It does not appear as an isolated component in the vector $z$, above). Hence errors in the input data $(x_i, y_i)$ result in an optimal choice for $(x_c, x_c^2 + r^2)$; rather than for $(x_c, r)$. Monte Carlo simulations have shown that this does not appear to have a significant effect, for expected noise conditions. In fact the simulations demonstrated a reduction in the standard deviation of the radial error by a factor of 2.5, compared to the standard deviation of the noise in the $(x_i, y_i)$ data points. This test was run for N=8 data points.