Filter Design by Pole/Zero Placement
For Band Pass and Notch Filters

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This type of filter is appropriate for high-Q band pass or notch filters. In this method, we will ‘throw darts at the Z plane’ to locate our poles and zeros. Filter parameters such as the amounts of attenuation or ripple in the stop band are not formal inputs to the design procedure. Rather these characteristics may be assessed after an initial pass through the design, and then adjusted (indirectly) via an iterative process.

Design Procedure:

1. (Filter Specification) Determine the center frequency, \( F_0 \), and the sample rate, \( F_s \). Also determine the pass band (or notch) width \( \Delta w \).
2. (Filter Design) Place poles and zeros. First, find the digital center frequency, \( w_0 \).
   a. (Notch) Define zeros at \( z_i = e^{j\omega_0} \) and poles at \( p_i = re^{j\omega_0} \)
   b. (Band pass) Define poles at \( p_i = re^{j\omega_0} \)

   Pole radius may be found via \( \Delta w \sim 2(1 – r) \), if \( r \sim 1 \).
3. (Filter Design) Setup \( H(z) \) in the form \( \frac{K_0(1 – z_i z^{-1})}{(1 – p_i z^{-1})} \).
   Where \( K_0 \) is a scaling factor, at your discretion. For pole radii, \( r \sim 1.0 \), try using \( K_0 = (1 – r) \), for band pass and \( K_0 = r \), for notch. These are reasonable estimates for a peak frequency response of \( \sim 1.0 \).
4. (Filter Design) Multiply factors in \( H(z) \) to identify \( a_k \) and \( b_k \) coefficients for the difference equation.
5. (Filter Analysis) Determine the frequency response and evaluate if it is satisfactory. If not, adjust poles and zeros in step 2, and repeat. The scaling factor in the numerator, \( K_0 \), may also be adjusted if the peak gain of the system is unsatisfactory.

A number of guidelines should be respected when locating poles and zeros:
- Place poles inside the unit circle, for stability.
- Use factors of the form \( (1 – re^{j\omega_0}z^{-1}) \), this yields a causal filter.
- When using complex poles and zeros, work with conjugate pairs, this results in real coefficients for your difference equation.