EE302 Analog Controls – Plotting a Root Locus - DePiero

This handout summarizes the steps needed to estimate a root locus plot for a system with \( T(s) = \frac{Y(s)}{R(s)} \), and with the forward and feedback blocks shown below. Several definitions precede the root locus procedure.

- Define \( GH(s) \) as the open loop transfer function. This assumes negative feedback. Also, note the \((-)\) of the negated input to the summer is not included in \( GH(s) \) for root locus.
- Define \( Nz \) and \( Np \) as the number of zeros and poles, respectively, for \( GH(s) \).
- Define \( \angle(s + z_i) \) as the angle of the vector difference between \( s \) and \(-z_i\), a zero of \( GH(s) \).
- Define \( \angle(s + p_j) \) as the angle of the vector difference between \( s \) and \(-p_j\), a pole of \( GH(s) \).
- Define the ‘Angle Criterion’ as \( \angle G(s)H(s) = 180^\circ \) or equivalent for a point \( s = s_0 \).

1. Plot the poles & zeros of \( GH(s) \) in the \( s \)-plane

2. Plot portions of real axis to the left of an odd number of poles and zeros.

3. The loci proceed from the poles to the zeros of \( GH(s) \). Zeros may be at infinity; in this case the loci proceed asymptotically. Find asymptote center \( \sigma_A \) and angles \( \phi_A \).

4. Use Routh-Hurwitz criteria to find the gain \( K \) at which the locus crosses the imaginary axis.

5. Find any breakaway (or break-in) points on the real axis.

6. Find the departure angles, \( \gamma_j \), for poles. Use angle criterion at a point \( s = s_0 \), infinitesimally close to the pole \( p_j \). Also find the arrival angles, \( \beta_j \), associated with zeros.

7. Estimate the path of the root locus.
   - \( Np \) gives the # of sections of the locus
   - Locus is symmetric with respect to the real axis in the \( s \)-plane.
   - Each section of the locus is continuous, moving from poles to zeros of \( GH(s) \).
   - Locus curves don’t generally cross

8. Find values of \( K_x \) at any points of interest, \( s = s_x \).

\[
GH(s) = \frac{\Pi_i(s + z_i)}{\Pi_j(s + p_j)}
\]

\[
\sigma_A = \left(\sum_j (-p_j) - \sum_i (-z_i)\right)/(Np - Nz)
\]

\[
\phi_A = 180^\circ (2q + 1)/(Np - Nz)
\]

\[
q = 0, 1, 2, ... Np - Nz - 1
\]

\[
\text{Find } s = s_0 \text{ where } \frac{d}{ds} \left\{ \frac{-1}{G(s)H(s)} \right\} = 0
\]

\[
\angle G(s)H(s) = 180^\circ \text{ or equivalent}
\]

\[
K_x = 1/|G(s_x)H(s_x)|
\]