Implementation, General Case

1) Update Kalman Gain matrix,
\[ K_k = P_k^{-} H_k^T (H_k P_k^{-} H_k^T + R_k)^{-1} \]

2) Acquire new measurement, \( z_k \)

3) Update State Estimate using new measurement,
\[ \hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-) \]

4) Use State Estimate, as needed in application

5) Update Error Covariance matrix,
\[ P_k = (I - K_k H_k) P_k^- \]

6) Project ahead. Find state based on system dynamics,
\[ \hat{x}_{k+1}^- = \phi_k \hat{x}_k \]

7) Project ahead. Find Error Covariance based on dynamics & expected process noise,
\[ P_{k+1}^- = \phi_k P_k^- \phi_k^T + Q_k \]

8) Advance discrete-time index \( k = k + 1 \)

9) Loop to step 1

Finding State Transition Matrix

Given system dynamics described by \( \dot{x} = F \dot{x} \) (continuous-time) the corresponding discrete-time state equation is \( x_{k+1} = \phi_k x_k + w_k \), and
\[ \phi_k = L^{-1}[(sI - F)^{-1}], \quad t = \Delta t \]

Where \( L^{-1}[] \) is the inverse Laplace Transform and \( \Delta t = \) sample period in seconds. The state vectors are Nx1 and the process noise for the discrete-time model, \( w_k \), is a white noise sequence. (Also see below).

Finding Error Covariance Matrix

Given a process noise source, \( f(t) \), and given transfer functions \( G_i(s) = X_i(s)/F(s) \) that relate \( f(t) \) to each of the components of the state vector \( x_i \). The Error Covariance matrix \( Q_k = E[w_k w_k^T] \) has \((i,j)\) elements
\[ E[x_i x_j] = \int_0^\Delta t \int_0^\Delta t g_j(\alpha) g_j(\beta) R_j(\alpha - \beta) d\alpha d\beta \]

Where \( g_i(\alpha) \) and \( G_i(s) \) are Laplace Transform pairs, and \( R_i(\alpha) \) is the autocorrelation function for the random process \( f(t) \).

Specific Implementation

- Position Measurements
- State: Position, Velocity, Acceleration
- Gaussian White Noise (Infinite Variance), PSD = \( W \)
- Sample Period = \( \Delta t \)

\[ \phi_k = \begin{bmatrix} 1 & \Delta t & \Delta t^2 / 2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \]

\[ Q_k = \begin{bmatrix} W \Delta t^5 / 20 & W \Delta t^4 / 8 & W \Delta t^3 / 6 \\ W \Delta t^4 / 20 & W \Delta t^3 / 3 & W \Delta t^2 / 2 \\ W \Delta t^3 / 6 & W \Delta t^2 / 2 & W \Delta t \end{bmatrix} \]