Phased Array Signal Processing
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Using phased array processing, signals from an array of detectors may be combined into a single output signal, in a manner that yields directional sensitivity for the system. This permits the system to receive (or transmit) signals in a particular spatial direction.

The technique is also referred to, as ‘beam forming’ because it is as if a narrow receptive conduit is established in some desired direction. This directional sensitivity can be described as a ‘pass direction’ (analogous to a ‘pass band’ in a filter).

The fundamental principle that provides the directional sensitivity is constructive (and destructive) interference. The processing algorithm described below combines the detector inputs and computes the total signal energy associated with source(s) located in some given direction, \( \theta \). This technique can be used to determine the true direction to a signal source, \( \theta \), by systematically varying \( \theta_a \) to find maximum signal strength – in which case \( \theta_a \) will equal \( \theta \). If \( \theta_a \) happens to equal \( \theta \) for some chosen \( \theta_a \), then constructive interference will be present in the beam forming process, which will yield a large output amplitude. If \( \theta_a \neq \theta \), then destructive interference reduces the amplitude.

**Array Geometry**

Where \( p_i(n) \) are the D detector signals, and \( x \) is the physical direction of the linear array. The detector spacing is assumed to be constant.
Processing Algorithm to Determine Direction to a Signal Source, $\theta_i$

1) Given an array of detector signals, $p_i(n)$.
2) Select a direction of interest $\theta_a$ ( $\theta_i$ is unknown).
3) For each of the D detector signals $p_i(n)$, $i = 0, D-1$
   a. Collect N samples of $p_i(n)$
   b. Transform sequence $p_i(n)$ to $P_i(k)$ via an N-point FFT
   c. Adjust phase of each component of $P_i(k)$ to yield $Q_i(k)$, using

\[
\Delta \phi_i^{\text{proc}}(k) = -2\pi \frac{\Delta x \cos(\theta_a)}{v} f_k i
\]

(See parameter table below.)

4) Add spectrum of all phase-adjusted signals, $Q_i(k)$, to yield $Q_{\text{sum}}(k)$

\[
Q_{\text{sum}}(k) = \sum_{i=0}^{D-1} Q_i(k), \text{ Where } Q_{\text{sum}}(k) \text{ is a spectrum with } N \text{ samples}
\]

5) Find total signal energy, $E_T$, in spectrum of combined signals

\[
E_T = \sum_{k=0}^{N-1} |Q_{\text{sum}}(k)|
\]

6) Output $E_T$ for the chosen angle $\theta_a$, i.e., $E_T = E_T(\theta_a)$.

7) Go to step 3, repeating for various angles $\theta_a$, and determine maximum value of $E_T$.

The angle $\theta_a$ associated with the maximum $E_T$ is assumed to equal $\theta_i$.

Parameter Description

| $\Delta x$ | Detector spacing (Meters) |
| $v$ | Velocity of wave propagation (Meters/Sec) |
| $f_k$ | Analog frequency of k-th spectral component (Hz) |
| $i$ | Detector index (dimensionless) |
| $\Delta \phi_i^{\text{proc}}(k)$ | Phase adjustment due to processing for k-th spectral component of i-th detector signal (Rads). This adjustment will align phase of i-th signal with the first detector, if $\theta_a = \theta_i$. |