(20) 1. The following relations describe a system. For each relation, state Y or N in the column to indicate its characteristic. Note that h(t) and s(t) are the step and impulse responses of a LTI system, respectively.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Time Invariant</th>
<th>Causal</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( h(t) = 2 e^{-4(t+1)} u(t+1) )</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>b) ( \frac{dy(t)}{dt} + 3 y(t) = 4 \frac{dx(t)}{dt} )</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>c) ( 2 y(t) = 4 x(t) + 5 )</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>d) ( \frac{dy(t)}{dt} + 3 t y(t) = \frac{1}{2} x(t+1) )</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>e) ( s(t) = 2 u(t-1) - 4 )</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
</tbody>
</table>

(20) 2. Given that the step response of the LTI system is \( s(t) = 4 \exp(-t) u(t) \), then determine the response \( y(t) \) to the input signal \( x(t) \) shown. Note that \( u(t) \) is the step function.

\[
\begin{align*}
\gamma(t) & = 2 u(t+1) - 2 u(t-3) \\
\Rightarrow \gamma(t) & = 2 \left[4 e^{-(t+1)} u(t+1)\right] - 2 \left[4 e^{-(t-3)} u(t-3)\right] \\
\gamma(t) & = 8 e^{-(t+1)} u(t+1) - 8 e^{-(t-3)} u(t-3) \\
\delta(t) & = \int_{-\infty}^{\infty} \lambda \gamma(\lambda) \delta(\lambda-t) \, d\lambda \\
& = 4 \delta(t+1) - 4 \int_{-\infty}^{\infty} \lambda \delta(\lambda-t) e^{-\lambda} u(t+1) \, d\lambda \\
& = 4 \delta(t+1) - 4 \int_{-\infty}^{\infty} \lambda \exp(-\lambda) u(t+1) \, d\lambda \\
& = 4 \delta(t+1) - 8 e^{-t} \int_{-\infty}^{t} \exp(-\lambda) u(t+1) \, d\lambda \\
& = 4 \delta(t+1) - 8 e^{-t} \left[ e^{-(t-\lambda)} u(t+1) - \phi(\lambda) \right] \\
& = 4 \delta(t+1) - 8 e^{-t} \left[ e^{-(t-\lambda)} u(t+1) - e^{-(t-3)} u(t+1) \right] \\
\end{align*}
\]
3. Given the following differential equation, initial conditions, and input for a LTI system:
\[ \frac{dy_1(t)}{dt} + 4y_1(t) = 2x(t) \quad \text{with} \quad y_1(0) = -2 \quad \text{and} \quad x(t) = u(t) \]

Showing all steps in your solution, find the responses indicated:

(7) a. zero input response, \( y_2(t) \)

\[ \frac{dy_2(t)}{dt} + 4y_2(t) = 0 \quad \text{with} \quad y_2(0) = -2 \]

i) homogeneous
\[ s + 4 = 0 \Rightarrow s = -4 \quad \therefore \quad y_2(t) = ke^{-4t} \]

ii) particular
\[ \frac{dy_2(t)}{dt} + 4y_2(t) = 0 \Rightarrow y_2(t) = M \Rightarrow 4M = 0 \Rightarrow M = 0 \Rightarrow y_2(t) = 0 \]

iii) complete
\[ y_2(t) = ke^{-4t} + 0 \quad \text{and} \quad y_2(0) = -2 \quad \therefore \quad k = \frac{1}{2} \]

\[ \therefore \quad y_2(t) = \frac{1}{2} e^{-4t} u(t) \]

(7) b. zero state response, \( y_3(t) \)

\[ \frac{dy_3(t)}{dt} + 4y_3(t) = 2x(t) \quad \text{with} \quad x(t) = u(t) \quad \text{and} \quad y_3(0) = 0 \]

solve for \( y_3(t) \) from
\[ \frac{dy_3(t)}{dt} + 4y_3(t) = 2(\frac{dx(t)}{dt}) + 2x(t) \quad \text{with} \quad y_3(0) = 0 \]

i) homogeneous
\[ s + 4 = 0 \Rightarrow s = -4 \quad \therefore \quad y_3(t) = Ke^{-4t} \]

ii) particular
\[ \frac{dy_3(t)}{dt} + 4y_3(t) = 2 \Rightarrow y_3(t) = M \Rightarrow 4M = 1 \Rightarrow M = \frac{1}{4} \]

\[ \therefore \quad y_3(t) = \frac{1}{4} e^{-4t} + \frac{1}{4} \]

\[ \Rightarrow \quad y_3(t) = -\frac{1}{2} e^{-4t} + \frac{1}{2} u(t) \]

(6) c. impulse response, \( h(t) \)

\[ \frac{dh(t)}{dt} + 4h(t) = 4x(t) \quad \text{with} \quad x(t) = 0 \quad \text{note:} \quad h_0(t) = 1 \quad \text{with} \quad n = 1 \]

solve for \( h(t) \) from
\[ \frac{dh(t)}{dt} + 4h(t) = 0 \quad \text{with} \quad h_0(0) = 1 \]

i) homogeneous
\[ s + 4 = 0 \Rightarrow s = -4 \quad \therefore \quad h_0(t) = Ke^{-4t} \]

ii) particular
\[ \frac{dh(t)}{dt} + 4h(t) = 0 \Rightarrow h(t) = M \Rightarrow 4M = 0 \Rightarrow M = 0 \]

\[ \therefore \quad h_0(t) = 0 \]

iii) complete
\[ h_0(t) = Ke^{-4t} + 0 = Ke^{-4t} \quad \text{and} \quad h_0(0) = 1 \Rightarrow k = 1 \]

\[ \therefore \quad h(t) = e^{-4t} \]

\[ \Rightarrow \quad h(t) = 2h_0(t) = 2e^{-4t} u(t) \]
4. Given \( h(t) = \text{rect}(t - \frac{1}{2}) \) and \( x(t) = 2u(t+1) - 2u(t-1) \), find \( y(t) \) and sketch it. Show all steps in your solution. Note that \( \text{rect}(t) \) is given in the figure shown.

\[
y(t) = x(t) \ast h(t) = \int_{-\infty}^{t} [2u(\lambda+1) - 2u(\lambda-1)] [u(t-\lambda) - u(t-\lambda-1)] d\lambda
\]

\[
= 2 \int_{-\infty}^{t} u(\lambda+1) u(t-\lambda) d\lambda - 2 \int_{-1}^{t} u(\lambda+1) u(t-\lambda-1) d\lambda + 2 \int_{0}^{t} u(\lambda-1) u(t+\lambda) d\lambda + 2 \int_{-1}^{0} u(\lambda-1) u(t+\lambda-1) d\lambda
\]

\[
y(t) = 2(t+1) u(t-1) - 2 u(t) - 2(t-1) u(t+1) + 2(t-2) u(t-2)
\]

\[
\text{pairwise sum } \{-1, 1\} + \{0, 1\} = \{-1, 0, 1, 2\} \quad \text{use } y(t) = \int_{-\infty}^{t} \frac{\chi}{\lambda} h(\lambda - x) d\lambda
\]

\[
t < -1 \implies y(t) = 0
\]

\[
-1 < t < 0 \implies y(t) = 2
\]

\[
0 < t < 1 \implies y(t) = 2(t+1)
\]

\[
1 < t < 2 \implies y(t) = 2
\]

\[
t > 2 \implies y(t) = 0
\]
5. Given the Fourier series \( x_p(t) = 2 + 2\cos(4\pi t) + 4\sin(8\pi t) \), showing all steps in your answer:

(a) determine fundamental frequency \( f_0 \) of \( x_p(t) \)

\[
f_0 = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 2 \quad \text{Hz}
\]

(c) sketch the one-sided spectrum, magnitude (\( C_k \)) and phase (\( \theta_k \)), for \( x_p(t) \).

6. Given the one-sided spectrum, magnitude and phase, shown in the figure, showing all steps in your answer, determine:

(a) the exponential form for the Fourier series of \( x(t) \)

\[
x_0(t) = 1 + 4e^{-j2\pi t} + 4e^{-j4\pi t} + 2e^{-j8\pi t}
\]

(b) the power \( P \) in the spectrum

\[
P = \sum_{k=-\infty}^{\infty} |C_k|^2 = 1^2 + 2^2 + 4^2 + 2^2 + 1^2 = 26 \quad \text{W}
\]