The exam will be closed book, no notes, and no calculator. It will cover the reading assignments in Chapters 3, 5 and 7, lecture material, and HW 1-3. Materials will not be covered on this exam. Paper will be provided; bring pencil and eraser.

Discrete Signals
representation: analytic \( x[n] = 2(\frac{1}{2})^n (u[n] - u[n-2]) \)
\( x[n] \) samples \( x[n] = 2 \delta[n] + \delta[n-1] \) \( (\sum_{k=-\infty}^{\infty} x[n]) \delta[n-k] \)

sequence \( x[n] = \left\{ \frac{2}{3}, 1 \right\} \) (remainder assumed zero)

energy: \( E = \frac{1}{2} \sum_{n=-\infty}^{\infty} |x[n]|^2 \) with \( \mathcal{E}(x[n]) = \frac{1}{\omega_0}, \mathcal{E}(x[n]) = \frac{1}{\frac{\omega_0}{2}} \), \( |x[n]| < 1 \)

signals: \( x[n], y[n], r[n], \cos(2\pi n F), \) with \( F = \frac{\omega_0}{2\pi} \) cycles/sample

operation on signals: shift, fold, scale

\( x[n] = \cos(2\pi n F) \) almost periodic if \( F = \frac{m}{N} \) (rational)

sampling of sinusoids \( x[n] = \cos(2\pi n F) = \sum_{k=-\infty}^{\infty} \cos[2\pi k N F] = \sum_{k=-\infty}^{\infty} \delta[n-k N] \), where \( F = \frac{m}{N} \)

and \( f_s = \frac{1}{T} \), sampling frequency

determination of common period of \( x[n] \) = \( \cos(2\pi n \frac{A_1}{N_1}) + \cos(2\pi n \frac{A_2}{N_2}) \), \( N = \text{LCM}(N_1, N_2) \)

decimation \( x[n] \) and interpolation \( x[n/N] \): to always recover \( x[n] \)

interpolate, then decimate

Discrete-Time Systems
linear operators: homogeneity, additivity, and superposition

\( \{a x[n] + b x[n]\} = a \{x[n]\} + b \{x[n]\} \)

time invariance: \( \{x[n]\} = y[n] \), then \( \{x[n-n]\} = y[n-n] \)

causal: \( y[n] \) does not depend on future inputs

static: \( y[n] \) depends only on \( x[n] \) (otherwise: dynamic (or memory)

stable: \( \text{BIBO} \), bounded input \( \Rightarrow \) bounded output

difference eq (LTI): \( y[n] + A_1 y[n-1] + \ldots + A_N y[n-N] = B_0 x[n] + \ldots + B_M x[n-M] \)

where \( M = N \), mathematical model for digital filter

FIR (finite impulse response): \( y[n] = B_0 x[n] + \ldots + B_M x[n-M] \), order \( M \)

IIR (infinite impulse response): \( y[n] = -A_1 y[n-1] - A_2 y[n-2] - \ldots - A_N y[n-N] + B_0 x[n] + \ldots + B_M x[n-M] \) (recursive), order \( N \)

realization: \( x[n] \) \[3^n \] delay \( x[n-0] \) \[ x[n] \] \[ A[n] \] \[ multipler \] \[ \Sigma \] \[ x[n] + y[n] \]

solution by recursion, use of initial conditions
analytic solution: homogeneous, particular, complete (with c.c.)
characteristic eq. \((1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}) y(n) = 0\)
\[ \Rightarrow 3^k + a_1 3^{k-1} + a_2 3^{k-2} + \cdots + a_n 3^0 = 0 \] solutions
\n\[ N \text{ roots } \Rightarrow y_k(z) = k_1 3^{n}_1 + k_2 3^{n}_2 + \cdots + k_N 3^{n}_N \]
particular solution: method of undetermined coefficients
\[ y_N(0) \text{ zero state }, i.e. y \equiv 0 \] use of \( y_N(z) \rightarrow B_0 y_N(0) + B_1 y_N(n-1) \)
y_{sec}(n) zero input with i.c.; \( y(n) = y_N(n) + y_{sec}(n) \)

impulse response: relaxed system for \( h(n) \)

recursion
\[ \text{FIR: } h(n) = B_0 s(n) + B_1 s(n-1) + \cdots + B_M s(n-M) \]
\[ \text{IIR: } h_0(n) \text{ with } i.c. = 0 \text{ and } h_0(0) = 1 \text{, then } \]
\[ h(n) = B_0 h_0(n) + B_1 h_0(n-1) + \cdots + B_M h_0(n-M) \]

stability BIBO \( \sum_{n=-\infty}^{\infty} |h(n)| < \infty \) (absolutely summable)

FIR: always stable
IIR: roots of char. eq. \( |\beta| < 1 \)
causal \( h(n) = 0 \text{ for } n < 0 \)

Discrete Convolution

relaxed system: zero state
linear convolution
\[ y(n) = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]
properties: commutes, linear, time-invariant, associative, distributive
\[ y[n] = [x[n] * h_1[n]] * h_2[n] = x[n] * [h_1[n] * h_2[n]] \]
where \( h_1[n] = h_1[n] + h_2[n] \)

\[ x[n] * \delta[n] = x[n] \]

finite sequences:
\[ y_{nc}(s) = h_1(s) + h_2(s), y_{nc}(e) = h_1(e) + h_2(e), y_{nc} = N_x + N_h - 1 \]
sum by columns
fold, shift, multiply
polynomial multiplication
zero padding: prepend, append, number constant
periodic input \( \Rightarrow \) periodic output in steady state (except first \( N \))

periodic convolution, \( N \)
\[ y_{pc}(n) = x_p(n) * h_p(n) \]
wrap-around for \( N \), use of zero padding
cyclic method
circulant matrix
zero padding to obtain linear convolution from periodic convolution