1. (10) The signal \( x(t) = 2\cos(16\pi t) \) is sampled at \( f_s = 10 \) Hz. Provide the Fourier transform of the sampled version of \( x(t) \), \( X(f) \), and draw the magnitude of \( X(f) \) from -20 to +20 Hz. Remember that \( g(t) = \cos(2\pi f_0 t) \) has the Fourier transform \( G(f) = 0.5\delta(f + f_0) + 0.5\delta(f - f_0) \).

\[
X(f) = 2g(t) = 2\cos(2\pi f_0 t) \leftrightarrow X(f) = \delta(f + 8) + \delta(f - 8)
\]

2. (10) \( X(f) \), the spectrum of \( x(t) \), is given below. If \( x(t) \) is sampled at a rate of \( f_s = 40 \) Hz, then draw the spectrum of \( X_s(f) \), the spectrum of the sampled signal \( x(t) \) on the figure below from -80 to 80 Hz.

\[
X_s(f) = \sum_{k=-\infty}^{\infty} X(f + kf_s) = \sum_{k=-\infty}^{\infty} \delta(f + kf_s)
\]

3. (10) Given a LTI system is modeled with difference equation \( y[n] + 0.5y[n-1] = 4x[n-1] \), then determine the frequency response function \( H_p(F) \) for the system.

\[
\begin{align*}
y_p(e^{j2\pi F}) &= \frac{Y_p(F)}{X_p(F)} \\
y_p(e^{j2\pi F}) &= \delta(\tau - 1) \leftrightarrow Y_p(\tau) = e^{-j2\pi \tau F} \\
\end{align*}
\]

\[
H_p(F) = \frac{4e^{-j2\pi F}}{1 + 0.5e^{-j2\pi F}}
\]
4. (10) Given $x[n] = \{1, 2, 1\}$, determine its DTFT, $X_P(F)$; show all steps in your solution.

\[ X_P(F) = \sum_{k=-\infty}^{\infty} x[k] e^{-j2\pi Fk} = \sum_{k=0}^{2} x[k] e^{-j2\pi Fk} \]

\[ = (1) e^{-j2\pi F(0)} + (2) e^{-j2\pi F(1)} + (1) e^{-j2\pi F(2)} \]

\[ = 1 + 2e^{-j2\pi F} + e^{-j4\pi F} \]

\[ \therefore X_P(F) = 1 + 2e^{-j2\pi F} + e^{-j4\pi F} \]

5. (10) Given that the DTFT $X_P(F) = 1 + 2e^{-j2\pi F} + e^{-j4\pi F}$, provide the magnitude and phase plots for one period, $0 < F < 1$. Show all work.

\[ X_P(F) = 1 + 2e^{-j2\pi F} + e^{-j4\pi F} \]

\[ = e^{-j2\pi F} \left( e^{j2\pi F} + 2 + e^{-j2\pi F} \right) \]

\[ X_P(F) = e^{-j2\pi F} \left( 2 + 2 \cos 2\pi F \right) \]

\[ \therefore |X_P(F)| = 2 + 2 \cos 2\pi F \]

\[ X - X_P(F) = -2\pi F \]
6. (10) For a LTI system with a frequency response function \( H_F(F) = \frac{4}{(1-e^{j8\pi F})} \) and a sinusoidal input \( x[n] = 2\cos(\pi/8)n \), determine the steady-state output for \( y[n] \); show all steps in your solution. [Note that this system is not BIBO stable, but does have a valid response for this input.]

\[
\begin{align*}
    x[n] &= 2\cos(\pi/8)n = 2\cos(2\pi/16)n \\
    y[n] &= 2|H_F(\frac{1}{16})|\cos(2\pi/16)n + \pi H_F(\frac{1}{16})
\end{align*}
\]

\[
\begin{align*}
    H_F(\frac{1}{16}) &= \frac{4}{1-e^{-j8\pi(\frac{1}{16})}} = \frac{4}{1-e^{-j\frac{\pi}{2}}} = \frac{4}{1-(-i)} = \frac{4}{1+i} \\
    H_F(\frac{1}{16}) &= \frac{4}{\sqrt{2} / 2} = 2\sqrt{2}/\sqrt{2} = \frac{2\sqrt{2}}{2}
\end{align*}
\]

\[
\begin{align*}
    y[n] &= 2\left(2\sqrt{2}\cos(\frac{\pi}{16}n - 45^\circ) \right) \\
    y[n] &= 4\sqrt{2}\cos\left(\frac{\pi}{8}n - 45^\circ\right)
\end{align*}
\]

7. Given \( x[n] = \{2, 0, 2\} \),

a) (4) determine \( X_{DFT}[k] \); show all steps in your solution.

\[
X_{DFT}(A) = \sum_{n=0}^{2} x[n] e^{-j2\pi\frac{k}{3}n} \\
= (2) e^{-j2\pi\frac{k}{3}(0)} + (2) e^{-j2\pi\frac{k}{3}(1)} + (2) e^{-j2\pi\frac{k}{3}(2)}
\]

\[
X_{DFT}(A) = 2 + 2 e^{-j\frac{2\pi}{3}} \Rightarrow X_{DFT}(A) = 2 + 2 e^{-j\frac{2\pi}{3}} = 2 - \sqrt{3} e^{-j\frac{\pi}{3}} = 2 - j\sqrt{3} = 2 - j60^\circ
\]

\[
X_{DFT}(B) = 2 + 2 e^{-j\frac{2\pi}{3}} = 2 + 2(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) = 1 + j\sqrt{3} = 2j60^\circ
\]

\[
X_{DFT}(C) = 2 + 2 e^{-j\frac{2\pi}{3}} = 2 + 2(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) = 1 - j\sqrt{3} = 2j(-60^\circ)
\]

b) (4) plot the magnitude and phase spectrum for your \( X_{DFT}[k] \).

c) (2) for your answer, show Parseval's theorem is satisfied

\[
\sum_{n=0}^{2} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X_{DFT}(k)|^2
\]

\[
2^2 + 0^2 + 2^2 = 8 = \frac{1}{3} \left[ 4^2 + 2^2 + 2^2 \right] = \frac{24}{3} = 8 \quad \checkmark
\]
8. Given \( x(t) = \cos(20\pi t) + \sin(30\pi t) \),
   a) (5) determine the minimum sampling frequency \( f_s \) for no aliasing
   \[
   \begin{align*}
   x(t) &= \cos(2\pi(10)t) + \sin(2\pi(15)t), \quad f_s = 10\text{ Hz}, \quad f_c = 15\text{ Hz} \\
   f_s &= 2f_{\text{max}} = 2(15) = 30\text{ Hz}
   \end{align*}
   \]
   b) (5) determine the minimum number of samples \( N \) for no leakage; show all work
   \[
   \begin{align*}
   f_s &= 30\text{ Hz} \\
   f_0 &= \gcd \{ 10, 15 \} = 5 \text{ Hz} \\
   \therefore \quad N &= f_s / f_0 = 30 / 5 = 6
   \end{align*}
   \]

9. Given the plot of \( X_{\text{DFT}}[k] \) below (the phase is zero for all values of \( k \)) and knowing that the sampling frequency of \( f_s = 36 \text{ Hz} \) is greater than the Nyquist frequency and there is no leakage, determine
   \[
   X_{\text{DFT}}[k]
   \]
   a) (5) the signal \( x[n] \)
   \[
   A \cos \frac{2\pi}{4} n \iff A \frac{N}{2} \sum_{k} \left[ A - k_0 \right] + A \frac{N}{2} \sum_{k} \left[ k - (N - k_0) \right]
   \]
   \[
   N = 4, \quad f_s = 36\text{ Hz}, \quad k_0 = 1, \quad N - k_0 = 4 - 1 = 3
   \]
   \[
   A \frac{N}{2} = 36 \quad \Rightarrow \quad A = 2(18) = 18
   \]
   \[
   \therefore \quad x(n) = 18 \cos \frac{2\pi}{4} n
   \]
   b) (5) the signal \( x(t) \)
   \[
   \frac{k_0}{N} = \frac{1}{3} \quad \Rightarrow \quad f_s = f_s \cdot \frac{k_0}{N} = 36 \cdot \frac{1}{3} = 9 \text{ Hz}
   \]
   \[
   \Rightarrow \quad x(t) = 18 \cos \frac{2\pi}{3} \cdot 9 t
   \]

10. (10) The following Matlab code script is the matlab2c.m file used in the homework. Provide the code lines that are indicated by the comments in the space allocated.

```matlab
%script for example 21.23  matlab2c.m
n = 0:50; %determine index n from 0 to 50
h=1:8;
y=conv(h,x); %set up first of two subplots
y=y(1:length(n));
sawtooth(2,t); stem(n,x); title('x(t)') xlabel('n') ylabel('y(t)') subplot(2,1,2) stem(n,y) title('truncated y(t)') xlabel('n') ylabel('y(t)')```