The goal of this project is to study the sampling of sinusoids, the relationship between the frequency (Hz) and the frequency index k and the period (number of samples N) and the sampling frequency fs, the effects of aliasing for fs < 2 fo (fo the largest frequency), and the effects of leakage due to ff/fs = k/N not being satisfied (where ff is the fundamental frequency for the periodic waveform if there is a sum of sinusoids). Use the code for the matlab4a.m file below, and enter three cases:

Case 1: fo = 10 Hz; fs = 40 Hz; N = 4
Case 2: fo = 10 Hz; fs = 15 Hz; N = 12
Case 3: fo = 10 Hz; fs = 15 Hz; N = 7

Observe the output angle(X) to understand the phase plot. For each of the three cases, annotate the plots of the DFT (FFT) to relate the index k to a frequency in Hz, and comment on the effects of aliasing and leakage if present. Comment the code to show that you understand its function, and enhance the code to improve the capability as your group sees fit. For your report, turn in the commented code, and the three annotated displays, and don’t forget to include the individual paragraphs for each team member on what was learned.

The report is due Wednesday 5/19/04

```
%matlab4a.m script for the study of sampling, aliasing,
%DFT, and leakage.
epsm=0.0001; %value for magnitude epsilon for zero value
epsa=0.0001; %value for angle epsilon (rad) for zero value
r='y'; %initial value for while loop
while(r=='y')
    f0=input('enter the value of f0 (Hz):');
    fs=input('enter the value of fs (Hz):');
    N=input('enter the number of samples N:');
    t=0:(N/fs)/200:N/fs;
x=cos(2*pi*f0*t);
subplot(4,1,1)
plot(t,x)
title('X(t)')
xlabel('t(s)')
nn=0:N;
xn=cos(2*pi*(f0/fs)*nn);
subplot(4,1,2)
stem(nn,xn)
title('X[n]')
xlabel('n')
nn=0:N-1;
xn=cos(2*pi*(f0/fs)*nn);
X=fft(xn,NN);
subplot(4,1,3)
stem(nn,abs(X))
title('magnitude of X')
xlabel('k')
Xk = abs(X); %logical vector for magnitude equal zero; true=1, false=0
Xk = unwrap(angle(Xk)); %replace small magnitude components with zero
Xk = abs(X)*epsa; %logical vector for angle equal zero; true=1, false=0
Xk = Xk.*Xk'; %replace small angle with zero
stem(nn,Xk)
title('phase of X')
xlabel('k')
r=input('Do you want try another set of inputs? y/n [n]:','s');
if isempty(r)
r='n';
end
if r=='n';
    return
end
```
case 1: $f_0 = 10 \, \text{Hz}$, $f_s = 40 \, \text{Hz}$, $N = 4$

\[ \frac{f_0}{f_s} = \frac{k_0}{N} \Rightarrow k_0 = \frac{f_0}{f_s} \cdot N = \frac{10}{40} \cdot 4 = 1 \]

$N - k_0 = 4 - 1 = 3$

Note: $f_s = 40 \, \text{Hz} \geq 2 \cdot (10 \, \text{Hz}) = 20 \, \text{Hz} \Rightarrow \text{no aliasing}$

$T_0 = \frac{1}{f_0} = 0.1 \, \text{s}$ and $T_{\text{dur}} = N \cdot \frac{1}{f_s} = 4 \cdot \frac{1}{40} = 0.1 \, \text{s} = T_0 = 0.1 \, \text{s}$

$\Rightarrow$ no leakage

\[ x(t) \]

\[ x(n) = \cos 2\pi \frac{f_0}{f_s} n = \cos 2\pi \frac{10}{40} n \quad \text{and} \quad \frac{f_0}{f_s} = \frac{k_0}{N} \Rightarrow \frac{10}{40} = \frac{2}{4} \Rightarrow k_0 = 1 \]

\[ x(n) = \cos 2\pi \frac{k_0}{N} n \Rightarrow \frac{N}{2} \delta_{n}[k - k_0] + \frac{N}{2} \delta_{n}[k - (N - k_0)] = x_{\text{det}}(k) \]

$\Rightarrow x_{\text{det}}(k) = 2 \delta_{n}[k-1] + 2 \delta_{n}[k-3]$
case 2: \( f_0 = 10 \, \text{Hz} \), \( f_s = 15 \, \text{Hz} \), \( N = 12 \)

\( f_s = 15 \, \text{Hz} < 2 (10 \, \text{Hz}) = 20 \, \text{Hz} \) \( \Rightarrow \) aliasing

\( f_a = f_s - f_0 = 15 - 10 = 5 \, \text{Hz} \) \( \Rightarrow \) \( \lambda a = \frac{f_a}{f_s} \cdot N = \frac{5}{15} \cdot 12 = 4 \)

\( \text{TDur} = N \cdot \lambda a = 12 \cdot \frac{4}{15} = 0.8 \, \text{s} = \frac{mT_0}{8} \cdot 0.1 = 0.85 \) \( \Rightarrow \) no leakage

\( x(t) \)

\( \lambda a \)

\( k \)

\( N - \lambda a \)

\( X[n] = \cos 2\pi \frac{\lambda a}{N} n = \cos 2\pi \frac{4}{12} n \)

\( X_{\text{DTFT}}[\lambda] = 6 \delta[n - 4] + 6 \delta[n - 8] \)

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Case 3: \( f_0 = \frac{10}{15} f_s \), \( f_s = 15 \text{ Hz} \), \( N = 7 \)

\[ f_s = 15 \text{ Hz} < 2 \text{a}\text{ Hz} \Rightarrow \text{same aliasing as case 2:} \quad f_a = \frac{5}{3} \text{ Hz} \]

\[ \frac{k_a}{N} = \frac{f_a}{f_s} \Rightarrow k_a = \frac{f_a}{f_s} N = 5 \]

\( \Rightarrow 3 \text{ spectral peak around } k = 2 \)

\( T_0 = \frac{1}{f_0} = 0.1 \text{ s} \) and \( T_{\text{dur}} = N \frac{1}{f_s} = \frac{7}{15} \Rightarrow \text{m} \quad T_0 = \text{m} \frac{1}{10} \)

\( \Rightarrow \text{leakage} \)

\[ x(t) \]

\[ 0 \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \quad 0.35 \quad 0.4 \quad 0.45 \quad 0.5 \]

\[ \text{magnitude of } X \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]

\[ 0 \quad -1 \quad -2 \quad -3 \quad -4 \quad -5 \quad -6 \quad -7 \]

\[ k \]

Since leakage \( \Rightarrow \) must use formula to get \( X_DFT(k) \)

\[ X_{DFT}(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{k}{N} n} \quad \text{with} \quad x(n) = \cos (2\pi \frac{f_0}{f_s} n) = \cos \frac{2\pi}{15} n \]

\[ X_{DFT}(k) = \sum_{n=0}^{6} \cos \frac{2\pi}{15} n e^{-j2\pi \frac{k}{7} n} \]