VECTOR IDENTITIES AND OPERATIONS

A.1 VECTOR IDENTITIES

The following vector identities may be helpful in manipulating Maxwell's equations and in solving for the electromagnetic field quantities. Instead of listing these identities in terms of E and B fields, the following table is prepared in terms of the general vectors A, B, and C.

\[
\begin{align*}
(A \times B) \cdot C &= (B \times C) \cdot A - (C \times A) \cdot B \\
A \times (B \times C) &= (A \cdot B) C - (A \cdot C) B \\
\nabla \cdot (\phi A) &= \nabla \phi \times A + \phi \nabla \cdot A, \quad \phi \text{ (Scalar)} \\
\nabla \cdot (A + B) &= \nabla \cdot A + \nabla \cdot B \\
\nabla \cdot (A \times B) &= B \cdot \nabla \times A - A \cdot \nabla \times B
\end{align*}
\]

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Sec. A.2 Vector Operations

\[
\begin{align*}
\nabla \cdot \nabla \phi &= \nabla \phi \\
\nabla \times \nabla \times A &= 0 \\
\nabla \times (\phi A) &= \nabla \phi \times A + \phi \nabla \times A \\
\nabla \times (A + B) &= \nabla \times A + \nabla \times B \\
\nabla \times (A \times B) &= A \nabla \cdot B - B \nabla \cdot A - (A \nabla) B \\
\n\nabla \cdot \nabla \phi &= 0 \\
\n\nabla \times A &= \nabla \times (\nabla \cdot A) - \nabla A \\
\int \nabla \cdot A \, d\mathbf{s} &= \int \nabla \cdot A \, d\mathbf{c} \quad \text{(Divergence theorem)} \\
\int \nabla \times A \cdot d\mathbf{s} &= \int \nabla \times A \cdot d\mathbf{c} \quad \text{(Stokes' theorem)}
\end{align*}
\]

A.2 VECTOR OPERATIONS

Divergence of a vector A. In the generalized curvilinear coordinate system, the divergence of the vector

\[
A = A_1 a_1 + A_2 a_2 + A_3 a_3
\]

is given by

\[
\text{div} A = \frac{1}{h_1 h_2 h_3} \left( \frac{\partial (h_2 h_3 A_1)}{\partial u_2} + \frac{\partial (h_1 h_3 A_2)}{\partial u_3} + \frac{\partial (h_1 h_2 A_3)}{\partial u_1} \right)
\]

The various components of A, the unit vectors \(a_1, a_2, a_3\), the independent variables \(u_1, u_2, u_3\), and the metric coefficients \(h_1, h_2, h_3\) in the Cartesian, cylindrical, and spherical coordinates are given by

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Cartesian</th>
<th>Cylindrical</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1, u_2, u_3)</td>
<td>(x, y, z)</td>
<td>(\rho, \phi, z)</td>
<td>(r, \theta, \phi)</td>
</tr>
</tbody>
</table>

| Vector components     | \(A_1, A_2, A_3\) | \(A_r, A_\theta, A_\phi\) | \(A_r, A_\theta, A_\phi\) |

| Unit vectors          | \(a_1, a_2, a_3\) | \(a_r, a_\theta, a_\phi\) | \(a_r, a_\theta, a_\phi\) |

| Metric coefficients   | 1, 1, 1        | \(1, \rho, 1\)            | \(1, r, r \sin \theta\) |