1. A lossless transmission line has a characteristic impedance $Z_0 = 100$ ohms and a velocity of propagation of $u_p = 3 \times 10^8$ m/s. The voltage on the transmission line is given by

$$v(z,t) = 8u(t-z/u_p) - 4u(t+z/u_p) \text{ (V)}$$

where $u(t)$ is the step function. Sketch the requested waveforms on the graphs below. Provide the scales for the voltage and current.

(10a) Provide a sketch of $v(1200m,t)$ on the graph below; note the units of $\mu$s on the time axis.

(10b) Provide a sketch of $i(z, 2\mu s)$, and give the scale for the current axis.
2. For the given lossless transmission line circuit, sketch the given voltage or current as specified on the graphs below, give the scale for axis of each.

(10a) \( v(600\text{m}, t) \)

\[ \begin{align*}
\frac{v}{u_0} &= \frac{9.33 \times 10^8}{3 \times 10^8}\% \\
v &= \frac{R_t - 2z}{R + 2z}
\end{align*} \]

(10b) \( i(z, 7\mu s) \)

\[ \begin{align*}
i^+ &= \frac{v}{R_L} + \frac{v}{R_6} \quad i^+ = 30 + (-10) = 20 \\
i^+ &= \frac{v}{R_L} + \frac{v}{R_6} \quad i^+ = 30 + (-10) = 20
\end{align*} \]
3. The phasor voltage for a lossless transmission line with \( Z_0 = 100 \) ohms is given by
\[
V(z) = \hat{V} e^{j\beta z} + \hat{V}^* e^{-j\beta z}
\]
Determine the following, giving units:
\[
\hat{V} = 100 \ e^{j\frac{\pi}{12}}
\]
\[
\frac{\beta}{\sqrt{\varepsilon_r}} = 0.02\ \text{in}
\]
\[
\beta = 1.26
\]
\[
2 = 2.6
\]

(2a) sinusoidal steady state voltage, \( V(z,t)_{ss} \)
\[
V(z,t)_{ss} = R_e \left\{ \hat{V} e^{j\omega t} \right\} = R_e \left\{ (\hat{V} e^{-j\beta z} + \hat{V}^* e^{j\beta z}) e^{j\omega t} \right\}, \ \omega = 2\pi f
\]
\[
V(0)_{ss} = 8 \cos \left( 2\pi (109)t - \pi / 3 \right) + 4 \cos \left( 2\pi (109)t + 3\pi / 2 \right)
\]

(2b) voltage reflection coefficient at \( z, \Gamma(z) \)
\[
\Gamma(z) = \frac{\hat{V} - e^{j\beta z}}{\hat{V} e^{j\beta z}} = \frac{4 e^{j3\pi/2} e^{-j3\pi/2}}{4 e^{j3\pi/2} e^{j3\pi/2}} = \frac{1}{2} e^{j(\pi/2 + \pi/3)} e^{j\pi/3} \quad \text{at} \quad \lambda = \lambda_{1/2}
\]
\[
\Gamma(0) = \frac{1}{2} e^{j5\pi/6}
\]

(2c) load voltage reflection coefficient, \( \Gamma_L \)
\[
\Gamma_L = \left| \Gamma(z) \right| = \frac{1}{2} e^{j5\pi/6}
\]
\[
|z| > 0
\]

(2d) voltage standing wave ratio, VSWR
\[
\Gamma_L = \rho e^{j\delta} = \frac{1}{2} e^{j5\pi/6}
\]
\[
\text{VSWR} = \frac{1 + \rho}{1 - \rho} = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = 3
\]

(4e) phasor \( \hat{I}(z) \)
\[
\hat{I}(z) = \frac{\hat{V} e^{-j\beta z}}{2z} + \frac{\hat{V} e^{j\beta z}}{-2z} = \frac{8 e^{-j\pi/3} e^{j3\pi/2}}{100} - \frac{4 e^{j3\pi/2} e^{-j3\pi/2}}{100}
\]
\[
= 80 e^{-j\pi/3} e^{-j3\pi/2} - 40 e^{j3\pi/2} e^{j\pi/3}
\]

(4f) total impedance of the line at \( z, \hat{Z}(z) \)
\[
\hat{Z}(z) = 2\pi \left[ 1 + \frac{\Gamma(z)}{1 - \Gamma(z)} \right] = 100 \left[ 1 + \frac{\frac{1}{2} e^{j5\pi/6} e^{-j5\pi/6}}{1 - \frac{1}{2} e^{j5\pi/6} e^{-j5\pi/6}} \right]
\]
\[
= 100 \left[ A - \sqrt{3} + \sqrt{3} \right] \left[ A + \sqrt{3} - \sqrt{3} \right]
\]
\[
= 100 \left( A - \sqrt{3} \right) \left( A + \sqrt{3} \right)
\]
\[
= 32 e^{j5\pi/6}
\]

(4g) load impedance, \( \hat{Z}_L \)
\[
\hat{Z}_L = \hat{Z}(z) = 2\pi \left[ 1 + \frac{\frac{1}{2} e^{j5\pi/6} e^{-j5\pi/6}}{1 - \frac{1}{2} e^{j5\pi/6} e^{-j5\pi/6}} \right] = 100 \left[ 1 + \frac{\frac{1}{2} (-\frac{\sqrt{3}}{2} + \sqrt{3} \frac{1}{2})}{1 - \frac{1}{2} (-\frac{\sqrt{3}}{2} + \sqrt{3} \frac{1}{2})} \right]
\]
\[
= 100 \left( A - \sqrt{3} \right) \left( A + \sqrt{3} \right)
\]
\[
= 32 e^{j5\pi/6}
\]

(4h) average power at the load, \( P_{ave}(t) \)
\[
P_{ave}(t) = \frac{1}{2} Re \left\{ \hat{V}(t) \hat{I}(t)^* \right\}
\]
\[
P_{ave}(0) = \frac{1}{2} Re \left\{ (8 e^{-j\pi/3} + 4 e^{j3\pi/2} \hat{I}(0)^*) \left( \hat{V}(0) + 4 e^{j3\pi/2} \hat{I}(0)^* \right) \right\}
\]
\[
= \frac{1}{200} Re \left\{ 64 - 32 e^{-j5\pi/6} + 32 e^{j3\pi/6} - 16 \right\}
\]
\[
= \frac{1}{200} Re \left\{ 48 - 32 \sqrt{3} \right\} = 0.240 \ (\omega)
\]
4. A transmission line with $Z_0 = 100 \, \Omega$ has the sinusoidal steady-state voltage
\[ v(z,t) = 8 \exp(-0.2z) \cos[3\pi(10^6)t + \pi/2 - 20\pi] + 4 \exp(0.2z) \cos[3\pi(10^6)t + \pi/4 + 20\pi] \] (V)
with $t$ in seconds and $z$ in meters. Give the values or expressions for the following, including their units:

(2) a) sinusoidal frequency
\[ \omega = 2\pi f = 3\pi(10^6) \Rightarrow f = 1.5 \, MHz \]

(2) b) phase constant
\[ \beta = 20\pi \, rad/m \]

(2) c) attenuation constant
\[ \alpha = 0.2 \, \mu \Omega/m \]

(2) d) wavelength
\[ \beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta} = 0.1 \, m \]

(4) d) phase velocity
\[ \beta = \frac{\omega}{\beta_p} \Rightarrow \beta_p = \frac{\omega}{\beta} = \frac{3\pi(10^6)}{20\pi} = 1.5 \times 10^5 \, m/s \]

(4) e) phasor for $v(z,t)$
\[ \hat{V}(z) = \hat{V} + e^{-j(0.2 + j20\pi)z} + 4e^{-j\frac{\pi}{4}} e^{j(0.2 + j20\pi)z} \] (v)

5. For the last two problems, use a Smith chart for each to present the solution; clearly annotate the Smith chart with sufficient detail so that a peer can understand how you arrived at each answer. It is expected that the answer value will be correct to within two significant digits, i.e., do not be too concerned with the accuracy of your answer, it is the procedure that is important. Your performance will be evaluated on the process presented to get the answer, and the construction on the Smith chart.

(10) a) Given the load impedance is $Z_L = 100 + j100$ for the following lossless transmission line, find the reflection coefficient at the input of the lossless transmission line, $\Gamma(-0.35\lambda)$

\[ \hat{Z}_L = \frac{Z_L}{Z_0} = \frac{100 + j100}{50} = 2 + j2 \] (A)

\[ \tan \phi = 0.35 \lambda \omega R_L + 0 \] (B)

\[ \Gamma(-0.35\lambda) = 0.62 / 137.5^\circ \]

(10) b) For the given lossless transmission line circuit, determine the input impedance at the input $Z_{in}$.

\[ \hat{Z}_{in} = \hat{Z}_L = \frac{Z_L}{Z_0} = \frac{50 - j50}{100} = \frac{1}{2} - j\frac{1}{2} \] (A)

\[ \hat{Y}_{in} = \hat{Y}_{in} = 1 + j \] (B)

rotate (B) 0.35\lambda \omega R_L + 0 (c)

\[ \hat{Y}_{in} = 0.98 \, \text{from} \, 0.35\lambda \omega R_L \]

\[ \text{from} \, \hat{Y}_{in} = 0.98 \, (s.c.) \]

\[ \hat{Y}_{in} = \hat{Y}_{in} + \hat{Y}_{in} = 0.9 + (0.8 + 0.8) = 0.9 \] (D)

\[ \hat{y}_{in} = \hat{y}_{in} + \hat{y}_{in} = 0.9 \, \text{and} \, 0.8 \] (E)

rotate (D) 0.8 \, \text{from} \, 0.35\lambda \omega R_L \]

Then, 180° - θ (F), which is same as (B)
The Complete Smith Chart
Black Magic Design

$\theta = 132.5^\circ$

$\frac{0.209}{0.589}$
Problem 5b)

The Complete Smith Chart

Black Magic Design

\[ \frac{0.161}{0.361} + 0.200 \epsilon \]

\[ z_m = 0.75 \]

\[ z_i = 25 - j \]

\[ 0.25 \lambda \]