1. The voltage on a transmission line with $Z_0 = 100$ ohms and $u_p = 2 \times 10^8$ m/s is given by
   
   $v(z,t) = 6 u(t - z/u_p) + 2 u(t + z/u_p)$ volts where $u(t)$ is the step function. Sketch the requested waveforms on the graphs below over the range shown.

   (6) a. $v(500\mu s)$

   $\frac{3}{u_p} = \frac{5\times10^8}{2\times10^8} = 2.5 \mu s$

   $v(500\mu s) = 6u(t-2.5\mu s) + 2u(t+2.5\mu s)$

   (6) b. $v(z,4\mu s)$

   $v(3.4\mu s) = 6u(4\mu s - \frac{3}{200}\mu s) + 2u(4\mu s + \frac{3}{200}\mu s)$

   (6) c. $i(z, -2\mu s)$ Provide the scale for the current axis.

   $i(3.3\mu s) = \frac{i^+(t-3/2\mu s) + i^-(t+3/2\mu s)}{2}$

   $= \frac{\frac{3}{100}u(-2\mu s - \frac{3}{200}\mu s)}{2}$

   $= \frac{3}{200}u(-2\mu s + \frac{3}{200}\mu s)$
2. For the following circuit with \( v_0(t) = 8 \ u(t) \) volts, sketch the following variables on the graphs over the range given:

\[ v(t) = -2 \ \text{V} \]

\[ i(t) = 2 \ \text{mA} \]

\[ g = 100 \ \text{m} \]

\[ f = 2 \ \text{MHz} \]

\[ R = 30 \ \text{k} \Omega \]

\[ C = 0.1 \ \mu \text{F} \]

\[ L = 2 \ \mu \text{H} \]

\( (10) \ a. \ v(100, t) \)

\[ \begin{align*}
\frac{1}{2} L \frac{d^2 i}{dt^2} + \frac{1}{C} \int i \ dt + \frac{1}{R} i &= 0 \\
\frac{1}{2} L \frac{d^2 i}{dt^2} &= -\frac{1}{C} \int i \ dt - \frac{1}{R} i
\end{align*} \]

\[ i(0) = i'(0) = 0 \]

\[ i(t) = \begin{cases} 
0 & \text{for } t < 0 \\
2 \ \text{mA} & \text{for } t \geq 0
\end{cases} \]

\[ V(t) = V_0 \ e^{-\frac{t}{\tau}} \]

\( \tau = \frac{1}{2} \ \mu \text{s} \)

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\( V(t) = \frac{1}{2} \ V_0 \ e^{-\frac{t}{\tau}} + \frac{1}{2} \ V_0 \ e^{-\frac{t}{\tau}}\]

\[ t = 0 \text{ s} \]

\[ t = 6 \text{ s} \]

\[ (10) \ b. \ i(z, 5 \mu s) \] provide the scale for current (don't forget units)

\[ i' = \frac{1}{\tau} = \frac{1}{2} \ \mu \text{s} = 50 \ \text{mA} \]

\[ \frac{1}{2} L \frac{d^2 i}{dt^2} = -\frac{1}{C} \int i \ dt - \frac{1}{R} i \]

\[ i(0) = i'(0) = 0 \]

\[ i(t) = \begin{cases} 
0 & \text{for } t < 0 \\
2 \ \text{mA} & \text{for } t \geq 0
\end{cases} \]

\[ V(t) = V_0 \ e^{-\frac{t}{\tau}} \]

\[ \tau = \frac{1}{2} \ \mu \text{s} \]

3. A transmission line with \( Z_0 = 50 \) ohms has the sinusoidal steady-state voltage

\[ v(z, t)_{ss} = 6 \ \text{exp}(-0.1z) \cos[6 \pi(10^6) t + \pi/4 - 40\pi z] \] (volts)

with \( t \) in seconds and \( z \) in meters. Give the values or expressions for the following, including their units:

(2) a. phase constant

\[ \beta = \frac{2\pi}{\lambda} \ (\text{rad/m}) \]

(2) b. attenuation constant

\[ \alpha = 0.1 \ (\text{Np/m}) \]

(2) c. wavelength

\[ \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2\pi/40} = 20 \ (\text{m}) \]

(2) d. phase velocity

\[ u_p = \frac{\omega}{\beta} = \frac{6\pi \times 10^9}{40\pi} = 1.5 \times 10^5 \ (\text{m/s}) \]

(2) e. phasor for \( v(z, t)_{ss} \)

\[ \hat{v} = \hat{V} e^{-j\frac{\beta z}{2}} \quad (v) \]
4. The phasor voltage for a lossless transmission line with \( Z_0 = 50 \) ohms is given by
\[
\hat{V}(z) = 4 e^{-j\pi/2} e^{j\pi z} + 2 e^{j\pi/2} e^{j\pi z} \text{ volts}
\]
The transmission line is terminated with a load impedance at \( z=0 \). Determine the following, giving units:

(4) a. phasor \( I(z) \)
\[
\hat{I}(z) = \frac{\hat{V} + e^{-j\pi/2}}{Z_0} - \frac{\hat{V} - e^{-j\pi/2}}{Z_0} = \frac{4 e^{-j\pi/2}}{50} e^{-j\pi z} - \frac{2 e^{j\pi/2}}{50} e^{j\pi z}
\]

(4) b. load voltage reflection coefficient, \( \Gamma_L \)
\[
\hat{\Gamma}_L = \frac{\hat{V}}{V_0} = \frac{2 e^{j\pi/2}}{4 e^{-j\pi/2}} = \frac{1}{2} e^{j\pi} = -\frac{1}{2}
\]

(4) c. voltage reflection coefficient at \( z, \hat{\Gamma}(z) \)
\[
\hat{\Gamma}(z) = \hat{\Gamma}_L e^{j\beta z} = (-\frac{1}{2}) e^{j2\pi z}
\]
with \( \beta = \pi \)

(4) d. voltage standing wave ratio, VSWR
\[
\hat{\Gamma}_L = \rho e^{j\theta} = \frac{1}{2} e^{j\pi} \Rightarrow \rho = \frac{1}{2}
\]
\[
\text{VSWR} = \frac{1 + \rho}{1 - \rho} = \frac{1 + (\frac{1}{2})}{1 - (\frac{1}{2})} = 3
\]

(4) e. load impedance, \( Z_L \)
\[
\hat{Z}_L(z) = Z_0 \frac{1 + \hat{\Gamma}(z)}{1 - \hat{\Gamma}(z)} \Rightarrow \hat{Z}_L(z) = 2_0 \frac{1 + \hat{\Gamma}(0)}{1 - \hat{\Gamma}(0)} = \frac{Z_0}{1 - \hat{\Gamma}(0)} = 50 \frac{1 + \frac{1}{2}}{1 - (\frac{1}{2})} = 50 \frac{1}{2} = \frac{50}{3} (\Omega)
\]

(4) f. average power at the load, \( P_{ave}(0) \)
\[
P_{ave}(z) = \frac{1}{2} \Re \left\{ \hat{V}(z) \hat{I}(z) \right\} = \frac{1}{2} \left| \hat{V} \right|^2 Z_0 \Re \hat{\Gamma}(z) \left( 1 - \left| \hat{\Gamma}(z) \right|^2 \right)
\]
\[
P_{ave}(0) = \frac{1}{2} \left| \frac{\hat{V}}{Z_0} \right|^2 Z_0 (1 - \left| \hat{\Gamma}(0) \right|^2) = \frac{1}{2} \left| \frac{\hat{V}}{Z_0} \right|^2 (1 - \left| \hat{\Gamma}(0) \right|^2)
\]
\[
= \frac{1}{2} \left( \frac{4}{50} \right)^2 (1 - (\frac{1}{2})^2) = \frac{16}{1800} (\frac{3}{4}) = \frac{12}{1000}
\]
\[
P_{ave}(0) = 0.12 \text{ (W)}
\]
(12) 5. Given the input impedance of a lossless transmission line is \( Z_m = 100 + j\, 100 \) ohms as shown below, then using the Smith chart on the next page, find the load impedance, \( Z_L \). Annotate the Smith chart with sufficient detail so that a peer can understand how you arrived at your answer. Your performance will be evaluated on the process presented to get the answer, and it is expected that the value will be correct within two significant digits, i.e., do not be too concerned over the accuracy of the construction of the answer.

\[ Z_m = 100 + j\, 100 \]

\[ Z_L = 0.25 \lambda \]

\[ \beta = 0 \]

(8) 6. Determine the electric field intensity \( \mathbf{E} \) at the origin for the configuration of two charges below; express your answer in terms of \( (4\pi\varepsilon_0) \), i.e., you don't have to evaluate it.

\[ \mathbf{E} = \frac{Q_i}{4\pi\varepsilon_0 r^2} \mathbf{a}_i \quad \text{linear} \Rightarrow \]

\[ \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{Q_1}{4\pi\varepsilon_0 r_1^2} \mathbf{a}_1 + \frac{Q_2}{4\pi\varepsilon_0 r_2^2} \mathbf{a}_2 \]

\[ \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \left( -\frac{4}{125} \frac{a_x}{a_y} - \frac{3}{125} \frac{a_y}{a_x} - \frac{2}{25} \frac{a_y}{a_y} \right) \]

(8) 7. Determine the magnetic flux density \( \mathbf{dB} \) at the origin for the configuration of the two differential current elements \( IDl \) as shown; express your answer in term of \((\mu_0/4\pi)\), i.e., you don’t have to evaluate it.

\[ \mathbf{B}_i = \frac{\mu_0}{2\pi} \frac{I \, dl \times Q_i}{r^2} \quad \text{linear} \]

\[ \mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \frac{\mu_0}{2\pi} \left( \frac{3 \, dl \times (-2\mathbf{a}_i)}{(3)^2} + \frac{\mu_0}{A} \frac{2 \, dl \times (-2\mathbf{a}_i)}{(2)^2} \right) \]

\[ \mathbf{B} = \frac{\mu_0}{2\pi} \left( \frac{3 \, dl \times (-2\mathbf{a}_i)}{3^2} + \frac{2 \, dl \times (-2\mathbf{a}_i)}{2^2} \right) \]

\[ = \frac{\mu_0}{2\pi} \left( -\frac{4}{3} \frac{a_x}{a_y} + \frac{8}{3} \frac{a_y}{a_y} \right) \]

\[ = \frac{\mu_0}{2\pi} \left( -\frac{4}{3} \frac{a_x}{a_y} + \frac{2}{2} \frac{2}{2} \right) \]
Problem 5

1. Normalize $\frac{z_{in}}{Z_0} = 100 + j100$
   
   $z_{in} = \frac{Z_{in}}{Z_0} = 1+j$

2. Plot $A$ \[ z(-0.25\lambda) = 1+j \]
   
   $\Pi(-0.25\lambda) = 0.45/60^\circ$

3. Rotate $\Pi(-0.25\lambda)$ 0.25\lambda
   
   WTL: 0.25\lambda = 0.16\lambda + x

   $x = 0.089 \implies \Pi(0) = 0.45/-116^\circ$
   
   Plot B

4. Read $z(0) = 0.5-j0.5$

5. Denormalize
   
   $z_L = z(0) = \frac{Z_0 z(0)}{Z_0 + z(0)}$
   
   $z_L = 50-150$Ω

Note:

\[ z(-\lambda) = z_0 \frac{1 + \Pi(-\lambda)}{1 - \Pi(-\lambda)} = z_0 \frac{1 + \frac{2z_0 z_L}{Z_0}}{1 - \frac{2z_0 z_L}{Z_0}} \]

or $z_L = Z_0 \frac{z(-\lambda)}{z_0}$