I have neither given nor received unpermitted aid during this examination.

Part I – closed book and notes, no calculator (30). When finished, turn in to get part II.

1. (10) State the units for the following electromagnetic field variables:

- Electric field intensity, $\mathbf{E}$: $\text{V/m}$
- Magnetic field intensity, $\mathbf{H}$: $\text{A/m}$
- Volume (free) charge density, $\rho$: $\text{C/m}^3$
- Conductivity, $\sigma$: $\text{S/m}$
- Permeability, $\mu$: $\text{H/m}$
- Electric flux density, $\mathbf{D}$: $\text{C/m}^2$
- Magnetic flux density, $\mathbf{B}$: $\text{Wb/m}^2$
- Permittivity, $\varepsilon$: $\text{F/m}$
- Poynting vector, $\mathbf{S}$: $\text{W/m}^2$

2. (4) For a linear isotropic material with $\sigma$, $\varepsilon$, $\mu$, and sources $\rho$ and $\mathbf{J}$, state the differential form in the time domain of the four Maxwell's equations that define the relationship between $\mathbf{E}$ and $\mathbf{H}$.

\[
\begin{align*}
\nabla \cdot \varepsilon \mathbf{E} &= \rho \\
\nabla \times \mu \mathbf{H} &= \mathbf{J} \\
\nabla \times \varepsilon \mathbf{E} &= -\varepsilon \frac{\partial \mathbf{H}}{\partial t} \\
\nabla \times \mu \mathbf{H} &= -\mu \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{J}
\end{align*}
\]

3. (3) For a linear isotropic material with $\sigma$, $\varepsilon$, $\mu$, state the constitutive relations.

\[
\begin{align*}
\mathbf{J} &= \sigma \mathbf{E} \\
\mathbf{D} &= \varepsilon \mathbf{E} \\
\mathbf{B} &= \mu \mathbf{H}
\end{align*}
\]

4. (3) For free space, state the value and units of the following:

- Permittivity, $\varepsilon_0$: $8.85 \times 10^{-12}$ $\text{F/m}$ or $\frac{1}{36\pi} \times 10^{-9}$ $\text{F/m}$
- Permeability, $\mu_0$: $4\pi \times 10^{-7}$ $\text{H/m}$
- Wave impedance, $\eta_0$: $377 \Omega$ or $120 \pi \Omega$

5. (4) State the four boundary conditions for two linear, isotropic materials with surface charge density $\rho_s$ ($\text{C/m}^2$) and surface current density $\mathbf{J}_s$ ($\text{A/m}$) as shown.

\[
\begin{align*}
\mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) &= \rho_s \\
\mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) &= 0 \\
\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) &= -\mathbf{J}_s
\end{align*}
\]

6. (6) Given the phasor representation of a uniform plane wave as shown, i.e., the phasor equations for linear polarized time harmonic TEM electric and magnetic field intensities in a linear isotropic homogeneous media:

\[
\begin{align*}
\hat{\mathbf{E}}_x(z) &\equiv (\hat{E}^{+}_m e^{j\beta z} + \hat{E}^{-}_m e^{-j\beta z}) \mathbf{a}_x (\text{V/m}) \\
\hat{\mathbf{H}}_y(z) &\equiv (\hat{H}^{+}_m e^{j\beta z} + \hat{H}^{-}_m e^{-j\beta z}) \mathbf{a}_y (\text{A/m})
\end{align*}
\]

giving units, define the following in terms of the variables in the phasor equations:

- Intrinsic (complex) wave impedance, $\eta$:
  \[
  \eta = \frac{\hat{E}^{+}_m}{\hat{H}^{+}_m} = -\frac{\hat{E}^{-}_m}{\hat{H}^{-}_m}
  \]

- Total field impedance at $z$, $Z(z)$:
  \[
  \hat{Z}(z) = \frac{\hat{E}_x(z)}{\hat{H}_y(z)}
  \]

- Reflection coefficient at $z$, $\Gamma(z)$:
  \[
  \hat{\Gamma}(z) = \frac{\hat{E}^{-}_m e^{j\beta z}}{\hat{E}^{+}_m e^{-j\beta z}} = \frac{\hat{E}^{-}_m}{\hat{E}^{+}_m} e^{j2\beta z}
  \]
1. Given an incident uniform plane wave in region 1 with phasor \( \hat{E}_1(z) = 2 \exp(-j2\pi z) \) \( \text{a}_x \) \( (\text{V/m}) = E_m^+e^{-j\beta_1z} \) is normally incident to the regions of perfect dielectrics, which all have \( \mu_0 \) and the \( \epsilon \) as shown. For the sinusoidal steady-state solution, using the smith chart on the next page determine the following. Provide your answers by annotating the smith chart for each part, and document each step in your solution; document your answers in enough detail so that a peer can understand your solution. Show all work. You must provide values for each part to get full credit, so be careful.

\[
0 = a(3) \hat{E}_3(O_2^+) = \frac{\hat{E}_{3m}^-}{\hat{E}_{3m}^+} = \frac{0}{\hat{E}_{3m}^+} = 0
\]

\[
\eta_0 = b(3) \hat{Z}_0(O_2^+) = \eta_0 \frac{1}{1 - \eta_0^2} = \eta_3 = \eta_0
\]

\[
0.6/180^\circ = c(3) \hat{Z}_2(O_2^-) = \frac{\eta_2}{\eta_3} = \frac{\eta_0}{\eta_3} \Leftrightarrow \eta_2 = 0.8\eta_0
\]

\[
0.12/180^\circ = d(3) \hat{F}_3(O_1^+) = 0.12 \angle 180^\circ
\]

\[
0.78/180^\circ = e(3) \hat{F}_3(O_1^-) = 0.78 \angle 180^\circ
\]

\[
i(2) \text{total phasor electric field intensity in region 1, } \hat{E}_1
\]

\[
\hat{E}_1 = \hat{E}_1^+ + \hat{E}_1^- = E_m e^{-j\beta_1z} \left( 1 + \frac{\beta_1}{\gamma_0} \right) \text{a}_x
\]

\[
j(2) \text{total phasor magnetic field intensity in region 1, } \hat{H}_1
\]

\[
\hat{H}_1 = \left[ \begin{array}{c} \hat{H}_1^+ e^{-j2\pi \gamma_0} \\ \gamma_0 \end{array} \right] = \left[ \begin{array}{c} \hat{H}_1^+ e^{j2\pi \gamma_0} \\ \gamma_0 \end{array} \right]
\]

\[
k(2) \text{P}_\text{av in region 1}
\]

\[
P_\text{av} = \frac{1}{2} \text{Re} \left\{ \hat{E}_1 \times \hat{H}_1 \right\} = \frac{|E_m|^2}{2\eta_1} \left( 1 - \gamma_0 \beta_1 \right)^2
\]
$\gamma_2(0^+_{\Omega}) = 1 + j \lambda = 4$
$\Gamma_1(0^+_{\Omega}) = 0.6 / 180^\circ$

$\lambda = 0.5 m \text{ with } d = 0.125$
$\Rightarrow \frac{0.125}{0.5} = 0.25 \lambda \text{ WTE}$

$\gamma_2(0^+_{\Omega}) = 0.25$
$\Gamma_1(0^+_{\Omega}) = 0.6 / 180^\circ$

$\gamma_2(0^-_{\Omega}) = \frac{\gamma_2(0^+_{\Omega})}{\eta_1} = \frac{4}{16}$
$\gamma_2(0^-_{\Omega}) = \frac{\gamma_2(0^-_{\Omega})}{\eta_1} = \frac{16}{16}$
$\gamma_3(0^-_{\Omega}) = \frac{\gamma_3(0^-_{\Omega})}{\eta_1} = \frac{(16/16)}{(16/2)}$
$\Gamma_1(0^-_{\Omega}) = 0.125$
$\Rightarrow \Gamma_1(0^-_{\Omega}) = 0.125 / 180^\circ$
2. The incident electric field intensity with $\theta_i = 45$ degrees is given by
\[ \mathbf{E}^i = (1.414 \mathbf{a}_x + 1.414 \mathbf{a}_y) \exp[-j2\pi(0.707x+0.707z)], \] (V/m)
with the media as shown, determine the following. Show all work. You must calculate the answer to get full credit, be so careful.

a. (3) Snell's law of reflection
\[ \Theta_r = \Theta_i = 45^\circ \]

b. (3) Snell's law of refraction
\[ \frac{\sin \Theta_i}{\sin \Theta_r} = \frac{\sqrt{\mu_r \varepsilon_r}}{\sqrt{\mu_i \varepsilon_i}} = \frac{1}{\sqrt{1.2}} \Rightarrow \sin \Theta_r = \sin \Theta_i \frac{1}{\sqrt{1.2}} = 0.508 \]

\[ \Theta_r = \sin^{-1} 0.508 = 30^\circ \]

\[ \Theta_i = \sin^{-1} 0.707 = 45^\circ \]

\[ \Theta_r = \sin^{-1} 0.508 = 30^\circ \]

\[ \Theta_i = \sin^{-1} 0.707 = 45^\circ \]

\[ \Theta_r = \sin^{-1} 0.508 = 30^\circ \]

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\[ \Theta_i = \sin^{-1} 0.707 = 45^\circ \]

\[ \Theta_r = \sin^{-1} 0.508 = 30^\circ \]

\[ \Theta_i = \sin^{-1} 0.707 = 45^\circ \]

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\[ \Theta_i = \sin^{-1} 0.707 = 45^\circ \]

\[ \Theta_r = \sin^{-1} 0.508 = 30^\circ \]

\[ \Theta_i = \sin^{-1} 0.707 = 45^\circ \]
3.(15) A TE\textsubscript{10} mode is propagating within the air-filled wave guide shown, and has the z component of the magnetic field intensity in the time domain given by
\[ H_z(x,y,z,t) = 4 \cos(10\pi x) \cos(\pi z) \cos(10\pi t) \text{ (mAm)} \]. Determine \( E_y(x,y,z,t) \) for \( x=5 \text{ cm} \), \( y=5 \text{ cm} \), \( z=1 \text{ m} \), and \( t=1.75 \text{ ns} \) using the procedure in problem 8.5 in hw8, i.e., the phasor of the electric field intensity for \( E_y(x,y,z) \) is given by
\[ \hat{E}_y(x,y,z) = \frac{\omega \mu}{(\alpha^2 + \omega^2 \mu^2)^{1/2}} \frac{\partial H_z(x,y,z)}{\partial y} = \frac{\omega \mu}{(\alpha^2 + \omega^2 \mu^2)^{1/2}} \frac{\partial E_y(x,y,z)}{\partial y} \]
Show all work and steps in your solution. Hint: note units of \( H_z \), and that \( \gamma^2 = -\beta^2 \).

\[ \hat{H}_z(x,y,z) = 4 \cos(10\pi x) \left\{ \frac{1}{x} e^{j10\pi z} \right\} \text{ and } \gamma = \frac{\beta}{c} = \frac{1}{10 \text{ cm}} \text{ (m/s)} \]

TE\textsubscript{10} mode \( \Rightarrow m=1 \), \( n=0 \) and \( \hat{E}_z = 0 \)

\[ \cos \frac{\pi}{\alpha} x = \cos \frac{\pi}{\alpha} x = \cos(10\pi x) \]

\[ \begin{align*}
\hat{E}_y(x,y,z) &= \frac{\omega \mu}{(10\pi)^2} \frac{\partial H_z(x,y,z)}{\partial y} = \frac{\omega \mu}{(10\pi)^2} \left[ 4 \cos(10\pi x) \cos(10\pi t) \right] \\
&= \frac{\omega \mu}{(10\pi)^2} \left( -40\pi x \cos^2 \frac{\pi}{\alpha} x \right) = \frac{\omega \mu}{(10\pi)^2} \left( -40\pi x \cos^2 \frac{\pi}{\alpha} x \right) e^{-10\pi t} \\
&= -10\pi x \cos(10\pi x) e^{-10\pi t} \text{ with } \gamma = e^{-j\frac{\pi}{2}} \\
\hat{E}_y(x,y,z) &= 0.960 \pi \text{ m} \cos(10\pi x) e^{-j(10\pi t + \pi/2)} \\
\end{align*} \]

\[ \begin{align*}
\hat{E}_y(x,y,z,t) &= \Re \{ \hat{E}_y e^{j\omega t} \} \\
E_y(x,y,z,t) &= 0.960 \pi \text{ m} \cos(6\pi x \cos^2 \frac{10\pi t}{2} + \pi/2) \\
E_y(0.05, 0.05, 1.75 \text{ ns}) &= 0.960 \pi \text{ m} \cos(6\pi (0.05)^2 + \pi/2) \\
&= 0.960 \pi \text{ m} \cos(6\pi (0.05)^2 + \pi/2) \\
&= 0.960 \pi \text{ m} \cos(0) \\
\end{align*} \]

\[ \begin{align*}
\hat{E}_y(0.05, 0.05, 1.75 \text{ ns}) &= 3.016 \text{ (V/m)} \\
\text{or} \ 1.005 \text{ (V/m)} \\
\end{align*} \]

\[ \text{Note: } \beta_0 = 10\pi \text{ in problem statement. I used } \lambda_0 = 0.2 \text{ m} \]

\[ \begin{align*}
\beta_0 &= \frac{2\pi}{\lambda_0} = 10\pi \text{, } \beta_0 \text{ should be } 10\pi \text{ (m/s)}. \\
\beta_0 &= 10\pi \text{ (m/s)}. \\
\end{align*} \]

\[ \text{Can you show that? Using } \frac{\gamma^2}{\beta^2 + \omega^2 \mu^2} = -\beta^2 + \omega^2 \mu^2 \text{ was acceptable for exam, though incorrect value, } \Rightarrow \frac{\omega \mu}{\gamma^2 + \omega^2 \mu^2} = 2\beta \]