I have neither given nor received unpermitted aid during this examination.

Part I — closed book and notes, no calculator (30). When finished, turn in to get part II.

1. (6) For free space, state the value and the units of the following:
   a. permittivity, \( \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \)
   b. permeability, \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)
   c. wave impedance, \( \eta_0 = \frac{377 \Omega}{2\pi} \)

2. (3) For a linear isotropic material with \( \sigma, \varepsilon, \mu \), state the constitutive relations.
   \[
   \mathbf{J} = \sigma \mathbf{E}, \quad \mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}
   \]

3. (12) For a linear isotropic material with \( \sigma, \varepsilon, \mu \), and sources \( \rho \) and \( \mathbf{J} \), state the four Maxwell's equations that define the relationship between \( \mathbf{E} \) and \( \mathbf{H} \): for:
   a. the differential form in time domain
   \[
   \nabla \cdot \varepsilon \mathbf{E} = \rho, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \cdot \mathbf{H} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J} + \sigma \mathbf{E}
   \]
   b. the integral form in the time domain
   \[
   \int_S \varepsilon \mathbf{E} \cdot d\mathbf{S} = \int_V \rho dV, \quad \int_S \mathbf{E} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \int_S \mathbf{H} \cdot d\mathbf{S}, \quad \int_S \mathbf{H} \cdot d\mathbf{S} = 0, \quad \int_S \mathbf{E} \cdot d\mathbf{S} = \int_V \mathbf{J} \cdot dV + \oint_S \mathbf{E} \cdot d\mathbf{S}
   \]
   c. the time harmonic form for phasors \( \mathbf{E} \) and \( \mathbf{H} \), and phasor sources \( \mathbf{\rho} \) and \( \mathbf{J} \):
   \[
   \nabla \cdot \varepsilon \mathbf{E} = \mathbf{\rho}, \quad \nabla \times \mathbf{E} = -j \omega \mathbf{H}, \quad \nabla \cdot \mathbf{H} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J} + j \omega \mathbf{E}
   \]

4. (6) For a homogeneous linear isotropic region which is source free, i.e., \( \rho = 0 \) and \( \mathbf{J} = 0 \), derive the Helmholtz equation using the vector identity \( \nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \), that is, showing all steps derive the source free wave equation for \( \mathbf{E} \) from the time harmonic form of Maxwell's equations.

5. (4) State the four boundary conditions for two linear, isotropic materials with surface charge density \( \rho_s \) (c/m²) and surface current density \( \mathbf{J}_s \) (A/m) as shown (note the regions carefully).

6. (3) Given equations for a uniform plane wave, i.e., the phasor equations for linear polarized time harmonic TEM electric and magnetic field intensities in a linear isotropic homogeneous media: \( \mathbf{E}_z(z) = (E_{z+} e^{j\beta z} + E_{z-} e^{-j\beta z}) \mathbf{a}_z \) (V/m) and \( \mathbf{H}_y(z) = (H_{y+} e^{j\beta z} + H_{y-} e^{-j\beta z}) \mathbf{a}_y \) (A/m), define the following in terms of the variables in the phasor equations, and give units:
   a. intrinsic (complex) wave impedance, \( \mathbf{Z} \)
   \[
   \mathbf{Z} = \frac{E_{z+}}{H_{y+}} = -\frac{E_{z-}}{H_{y-}} \quad (\Omega)
   \]
   b. total field impedance at \( z \), \( \mathbf{Z}(z) \)
   \[
   \mathbf{Z}(z) = \frac{E_{z+}(z)}{H_{y+}(z)} \quad (\Omega)
   \]
   c. reflection coefficient at \( z \), \( \Gamma(z) \)
   \[
   |\Gamma(z)| = \frac{E_{z-}(z)}{E_{z+}(z)} = \frac{1}{\mathbf{Z}(z)}
   \]
1. (28) Given an incident uniform plane wave in region 1 with phasor $E_1(z) = 2 \exp(-j4\pi z) a_x$ (V/m) is normally incident to the regions of perfect dielectrics, which all have $\mu_0$ and the $\varepsilon$ as shown. Using the smith chart on the next page determine the sinusoidal steady-state solution for the total electric field intensity $E(0^-)$, i.e., at the boundary in region 1. Document each step in your solution, and annotate the smith chart in enough detail so that a peer can understand your solution. Show all work.

\[ \Gamma_3(\theta) = 0 \Rightarrow \Gamma_3(\theta^*) = 0 \]

\[ Z_3(\theta^*) = \eta_3 \frac{1 + \Gamma_3(\theta^*)}{1 - \Gamma_3(\theta^*)} = \eta_3 = \eta_0 \]

\[ Z_3(\theta^*) = \frac{Z_3(\theta)}{\eta_0} = \frac{\eta_0}{\eta_0} = 2 \]

Plot on Smith chart: $Z(\theta^*)$

reflect $\frac{0.05 m}{0.25 m} = 0.2 \lambda$ w/ $\theta^*$

$\theta_0 \Gamma_2(\theta^*) = 0.34 \angle -144^\circ$

$\Rightarrow Z_2(\theta^*) = 0.54 - j0.24$

\[ Z_2(\theta^*) = \eta_2 Z_2(\theta^*) = \frac{Z_2(\theta)}{\eta_2} \]

\[ \Rightarrow \Gamma_2(\theta) = \frac{-Z_2(\theta^*)}{\eta_2} = \frac{0.54 - j0.24}{\eta_2} = 0.27 - j0.12 \]

\[ \Rightarrow \lambda_2 = \frac{0.27}{0.12} = 2.25 \angle -30^\circ \]

\[ \hat{E}(\theta) = \hat{E}_m e^{-j\beta_1 z} (1 + \Gamma_1(\theta)) \hat{a}_x = \hat{E}_m (1 + \Gamma_1(\theta^*)) \hat{a}_x \]

$\Rightarrow \hat{E}(\theta) = 0.93 \angle -18.9^\circ \hat{a}_x (V/m)$
The Complete Smith Chart
Black Magic Design

$\Gamma_{2}(0^\circ) = 0.34 / 180^\circ$

$\Gamma_{1}(0^\circ) = 0.58 / -180^\circ$

$\Gamma_{2}(0^\circ) = 0.2 \lambda_{WS}$
2. A uniform plane wave is normally incident to a boundary at \( z = 0 \) between two perfect dielectrics and it is determined that the reflection coefficient at a location in the region of the incident field is given by \( \Gamma(z) = 0.50 \exp(j8\pi z) \). If the phasor for the incident electric field intensity is given by \( \vec{E}'(z) = 2 \exp(-j4\pi z) a_x \) (V/m) in the region \( z < 0 \) with \( \epsilon_r = 4 \) and \( \mu_0 \), then at the location \( z = -1 \) (m), determine:

a.(4) the total phasor electric field intensity \( \vec{E}(-1) \)

\[
\begin{align*}
\vec{E}(3) &= \vec{E}'(3) + \vec{E}^r(3) \\
\vec{E}(3) &= \vec{E}'(3) (1 + \Gamma(3)) \\
\vec{E}(-1) &= 2 e^{-j4\pi z} (1 + 0.50 e^{-j8\pi z}) a_x \\
\vec{E}(-1) &= 2 e^{-j4\pi} (1 + 0.50 e^{-j8\pi}) a_x \\
\vec{E}(-1) &= 3 a_x \ (\text{V/m})
\end{align*}
\]

b.(4) the total phasor magnetic field intensity \( \vec{H}(-1) \)

\[
\begin{align*}
\vec{H}(3) &= \vec{H}'(3) + \vec{H}^r(3) \quad \text{with} \quad \eta_l = \sqrt{\frac{\mu_0}{\epsilon_1}} = \sqrt{\frac{4\pi \mu_0}{2\epsilon_0}} = \frac{\eta_0}{\sqrt{2}} = 60\pi \\
\vec{H}'(3) &= \frac{\vec{E}'(3)}{\eta_l} e^{-j4\pi z} a_y \\
\vec{H}^r(-1) &= \frac{2}{60\pi} e^{-j4\pi} a_y = \frac{1}{30\pi} e^{-j4\pi} a_y = 0.0106 a_y \\
\vec{H}(3) &= -\frac{\vec{E}(3)}{\eta_l} e^{j4\pi z} a_y = \frac{-\Gamma(3)}{\eta_l} e^{j4\pi z} a_y \\
\vec{H}(-1) &= -\frac{(0.50)(2)}{60\pi} e^{j4\pi} a_y = -\frac{1}{60\pi} e^{-j4\pi} a_y = -\frac{1}{60\pi} a_y \\
\vec{H}(-1) &= \frac{1}{60\pi} a_x - \frac{1}{60\pi} a_y \ (\text{A/m}) = 0.0053 a_y \ (\text{A/m})
\end{align*}
\]

c.(4) the average power density \( P_{ave}(-1) \)

\[
\begin{align*}
P_{ave}^c(-1) &= \frac{1}{2} \Re \{ \vec{E}'(3) \times \vec{H}^r(3) \} = \frac{1}{2} \Re \left\{ 2a_x \times \left( \frac{1}{30\pi} a_y \right) \right\} = \frac{1}{30\pi} a_x^2 \\
P_{ave}^r(-1) &= \frac{1}{2} \Re \{ \vec{E}^r(-1) \times \vec{H}^r(-1) \} = \frac{1}{2} \Re \left\{ 1 a_x \times \left( -\frac{1}{60\pi} a_y \right) \right\} = -\frac{1}{120\pi} a_x^2 \\
P_{ave}(-1) &= P_{ave}^c + P_{ave}^r = \frac{1}{30\pi} a_x^2 + \left( -\frac{1}{120\pi} a_x^2 \right) = \frac{1}{40\pi} a_x^2 \\
P_{ave}(-1) &= 0.00796 a_x^2 \ (\text{W/m}^2)
\end{align*}
\]
3. The phasor representation of the electric field intensity of an uniform plane wave incident to a boundary of two dielectrics as shown is given by
\[ \mathbf{E}' = (2 \mathbf{a}_x - 3.464 \mathbf{a}_y) \exp[-j 4\pi (0.866x + 0.5z)] \text{ (V/m)} \]
In region 1, determine the following:
(a) The total phasor electric field intensity, \( \mathbf{E}_1 \)
\[
\frac{\sin \Theta_i}{\sin \Theta_k} = \frac{E_i}{E_k} = \frac{\sqrt{E_i^2}}{\sqrt{E_k^2}} = \frac{\sqrt{4.60}}{\sqrt{2}} = 2
\]
\[ \Theta_k = \sin^{-1} \left( \frac{1}{2} \sin \Theta_i \right) = \sin^{-1} \left( \frac{1}{2} \sin 60^\circ \right) = 25.66^\circ \]
\[ \Gamma_{ii} = -\frac{\hat{E}_r^r}{\hat{E}_r^i} = \frac{\eta_1 \cos \Theta_i - \eta_1 \cos \Theta_k}{\eta_1 \cos \Theta_k + \eta_1 \cos \Theta_i} \]
\[ \Gamma_{ii} = \frac{\eta_0}{2} \cos 25.66^\circ - \eta_0 \cos 60^\circ = -0.052 \]

Now \( \beta_i = 4\pi \sqrt{(2)^2 + (-3.464)^2} = 4 \) with \( \hat{E}_r^i = -\Gamma_{ii} \hat{E}_r^i \) \( |\beta_r| = \sqrt{\frac{\eta_0}{2} / \frac{\eta_0}{2} \cos 25.66^\circ + \eta_0 \cos 60^\circ} = 4\pi(0.866a_x - 0.5a_y) \)
\[ \hat{E}_r^r = -\Gamma_{ii} \hat{E}_r^i (a_x \sin \Theta_i - a_y \cos \Theta_i) e^{-j \beta_r r} \]
\[ \hat{E}_r^r = -(-0.052)(4)(-0.5a_x - 0.866a_y) e^{-j \beta_r r} \]
\[ \hat{E}_i = \hat{E}_r^i + \hat{E}_r^r \]
\[ = (2a_x - 3.464a_y) e^{-j \beta_r r} \]
\[ + (-0.104a_x - 0.18a_y) e^{-j \beta_r r} \]

(b) The reflected magnetic field intensity, \( \mathbf{H}_r \)
\[ \mathbf{H}_r = \mathbf{H}_r^r + \mathbf{H}_r^i \]
\[ \mathbf{H}_r^r = \mathbf{n} \mathbf{E}_r^r / \eta_0 = \frac{1}{\eta_0} \begin{vmatrix} a_x & a_y & a_z \end{vmatrix} \]
\[ = \frac{-a_y}{\eta_0} \left[ (0.866)(-0.180) - (-0.5)(-0.104) \right] e^{-j \beta_r r} \]
\[ \mathbf{H}_r^i = 0.55 a_y e^{-j \beta_r r} \]
\[ (0.21) \]
4. The phasor representation of the electric field intensity of an uniform plane wave incident to a boundary of two dielectrics as shown is given by \( \vec{E} = (4a_2) \exp[-j4\pi(0.5x+0.866z)] \) (V/m).

In region 2, determine the following:

a. (9) the total phasor electric field intensity, \( \vec{E}_2 \)

\[ \vec{E}_2 = \vec{E}_2^i \perp \text{plane of incidence} \]

For perpendicular case with \( \Theta = \Theta_e = 60^\circ \)

\[ \frac{\sin \Theta_e}{\sin \Theta_i} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{4\varepsilon_2}{\varepsilon_0}} = 2 \]

\[ \Theta_e = \sin^{-1} \left( \frac{1}{2} \sin \Theta_i \right) = \sin^{-1} \left( \frac{1}{2} \sin 30^\circ \right) = 14.4^\circ \]

\[ \tau_{\perp} = \frac{2 \varepsilon_2 \cos \Theta_e}{\varepsilon_2 \cos \Theta_i + \varepsilon_1 \cos \Theta_e} = \frac{2 \left( \frac{1}{2} \right) \cos 30^\circ}{\varepsilon_2 \cos 30^\circ + \varepsilon_1 \cos 14.4^\circ} \]

\[ \tau_{\perp} = 0.62 \]

\[ \vec{E}_{2i} = \tau_{\perp} \vec{E}_1 \]

And \( \beta_e = \beta_e \left( \sin \Theta_e \frac{a_2}{a_1} + \cos \Theta_e \frac{a_3}{a_1} \right) \)

\[ = 8\pi \left( \sin 14.4^\circ \frac{0.25}{0.4} + \cos 14.4^\circ \frac{0.97}{0.4} \right) \]

\[ = 8\pi \left( 0.25 \times 0.97 + 0.97 \frac{2}{2} \right) \]

\[ \vec{E}_2 = \vec{E}_{2i} = (0.62)(4) a_1 e^{-j\beta_e \cdot 5} \]

\[ \vec{E}_2 = 2.48 a_1 e^{-j\beta_e \left( 0.25x + 0.97z \right)} \] (V/m)

b. (6) the total phasor magnetic field intensity, \( \vec{H}_2 \)

\[ \vec{H}_2 = \frac{\eta \beta_e \times \vec{E}_2}{\eta_2} = \frac{1}{\eta_2} \begin{vmatrix} a_2 & a_1 & a_3 \\ 0.25 & 0 & 0.97 \\ 0 & 2.48 & 0 \end{vmatrix} e^{-j\beta_e \cdot 5} \]

\[ = \frac{e^{-j\beta_e \cdot 5}}{\eta_2} \begin{bmatrix} -a_2(0.97)(2.48) + a_3(0.25)(2.48) \\ -a_2(0.25)(2.48) + a_3(0.97)(2.48) \\ -a_2(0.25)(0.97) + a_3(0.25)(0.97) \end{bmatrix} \]

\[ = (-12.9 a_2 + 3.3 a_3) e^{-j\beta_e \left( 0.25x + 0.97z \right)} \] (mAm/m)

\[ \beta_{\perp} = 2 \beta_0 \]

\[ \beta_{\parallel} = 8\pi \left( \cos \beta_0 \right) \]